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PHYS 600 – HW1 SOLUTIONS

PROBLEM 1 (*a*) let $\vec{a}$ and $\vec{b}$ denote two adjacent rhombus sides; one of the diagonals is then $\vec{a}+\vec{b}$ and the other is $\vec{a}-\vec{b}$. Taking the scalar product between the 2 diagonals we will obtain that $\left(\vec{a}-\vec{b}\right).\left(\vec{a}+\vec{b}\right)=\vec{a}^{2}-\vec{b}^{2}=0$

because the magnitudes of $\vec{a}$ and $\vec{b}$ are equal (rhombus sides). It results that the diagonals are perpendicular to each other

PROBLEM 2 (*a*) the velocity is obtained by differentiating $\vec{r}=\hat{x}\,A\cos(\omega t)+\hat{y}\,A\sin(\omega t)$:

$$\vec{v}=\omega A\,(-\hat{x}\sin(\omega t)+\hat{y}\cos(\omega t))$$

It is easily verified that the scalar product $\vec{r}.\vec{v}=0$ so $\vec{r}$ and $\vec{v}$ are perpendicular to each other

(*b*) the acceleration is obtained by differentiating the velocity : $\vec{a}=-\omega^2 A\,(\hat{x}\cos(\omega t)+\hat{y}\cos(\omega t))=-\omega^2\,\vec{r}$

(*c*) the angular momentum is equal to : $\vec{L}=\vec{r}\times m\vec{v}=(\hat{x}\,A\cos(\omega t)+\hat{y}\,A\sin(\omega t))\times m\,\omega A\,(-\hat{x}\sin(\omega t)+\hat{y}\cos(\omega t))=$

$=m\,\omega A^2(\cos^2(\omega t)+\sin^2(\omega t))\,\hat{z}=m\,\omega A^2\,\hat{z}$. Its magnitude is : $\left|\vec{L}\right|=m\,\omega A^2$

PROBLEM 3 (*a*) $\vec{\nabla}.\vec{E}=-\mu^2\phi$ and $\vec{\nabla}\times\vec{H}=\partial_t\vec{E}-\mu^2\vec{A}$

so $\partial_t\,\vec{\nabla}.\vec{E}=-\mu^2\,\partial_t\,\phi$ and $\vec{\nabla}.\left(\vec{\nabla}\times\vec{H}\right)=\vec{\nabla}.\,\partial_t\,\vec{E}-\mu^2\,\vec{\nabla}.\vec{A}$

Adding the last 2 equalities and using that $\vec{\nabla}.\left(\vec{\nabla}\times\vec{H}\right)=0$ and $\partial_t\,\vec{\nabla}.\vec{E}=\vec{\nabla}.\,\partial_t\,\vec{E}$,

we remain with $0=-\mu^2\left(\partial_t\,\phi+\vec{\nabla}.\vec{A}\right)$

(*b*) taking the divergence of the equation : $\vec{E}=-\partial_t\,\vec{A}-\vec{\nabla}\,\phi$ it results $\vec{\nabla}.\vec{E}=-\partial_t\,\vec{\nabla}.\vec{A}-\nabla^2\,\phi$

Replace now $\vec{\nabla}.\vec{A}$ by $-\partial_t\,\phi$ (see part (*a*)) and $\vec{\nabla}.\vec{E}$ by $-\mu^2\,\phi$ to obtain : $(\nabla^2-\partial_t^{\,2}-\mu^2)\,\phi=0$

(*c*) evaluate the integral of the vector $\phi\,\vec{\nabla}\,\phi$ over the closed surface S containing the domain V :

$$0=\int_S\phi\,\vec{\nabla}\,\phi\,d\vec{S}=\int_V\vec{\nabla}\left(\phi\,\vec{\nabla}\,\phi\right)d^3x=\int_V\left(\left(\vec{\nabla}\,\phi\right)^2+\phi\,\nabla^2\phi\right)d^3x=\int_V\left(\left(\vec{\nabla}\,\phi\right)^2+\mu^2\,\phi^2\right)d^3x\ \geq 0$$

The equality is possible only if $\phi=0$ inside V

PROBLEM 4 consider a rotation R given in matrix form by R_{ij} such that

$\hat{x}_i'=R_{ij}\,\hat{x}_j$ (basis transformation) and $x_i'=R_{ij}\,x_j$ (coordinates transformation).Then $x_i=R_{ji}\,x_j'$ because $R^{-1}=R^t$

Applying the differentiation chain rule we can obtain the transformation of the partial derivatives

$\partial_i\,'=\dfrac{\partial\,x_k}{\partial\,x_i'}\,\partial_k=R_{ik}\,\partial_k$ .It results that the transformed operator will be :

$$\vec{\nabla}\,'=\hat{x}_i'\,\partial_i\,'=R_{ij}\,\hat{x}_j\,R_{ik}\,\partial_k=\delta_{jk}\,\hat{x}_j\,\partial_k=\hat{x}_j\,\partial_j=\vec{\nabla}$$