MIDTERM EXAMINATION PHYSICS 600 October 19, 2004

*indicates a homework problem.

(25pts) [1]* Prove the following properties of the Dirac δ function

a) $\delta(x^2 - a^2) = \frac{1}{2|a|}[\delta(x + a) + \delta(x - a)]$

b) $x\delta'(x) = -\delta(x)$

c) $\delta[g(x)] = \sum_i \frac{\delta(x - x_i)}{|[\partial g/\partial x]_{x=x_i}|}; g(x_i) = 0$

(25pts) [2] Consider the tensor $T^{\lambda}_{\mu\nu}(x')$.

a) What would the components of this tensor be in another coordinate system x?

b) Write down the expressions for the covariant derivative of a covariant vector V_{μ}, and for a contravariant vector W^{λ}. (You need not derive these expressions.)

c) Use the results in part b) to derive the expression for the covariant derivative of $T^{\lambda}_{\mu\nu}(x)$.

d) How many independent components would $T^{\lambda}_{\mu\nu}$ have in 3 dimensions, assuming no particular symmetry of the indices?

e) How many independent components would it have if $T^{\lambda}_{\mu\nu} = T^{\lambda}_{\nu\mu}$?

f) How many components would it have in 3 dimensions if $T^{\lambda}_{\mu\nu} = T^{\lambda}_{\nu\mu}$?

(25pts) [3] a) Prove that every n-dimensional vector space V over a field F is isomorphic to the space F^n of all n-tuples of F. (Hint: Begin by stating **precisely** what the isomorphism is.)

b) In the vector space of real n-tuples prove that the following n-vectors are linearly independent:

$$x_1 = (1, 1, ..., 1, 1)$$
$$x_2 = (0, 1, ..., 1, 1)$$
$$x_3 = (0, 0, ..., 1, 1)$$
$$\vdots$$
$$x_n = (0, 0, ..., 0, 1)$$

(25pts) [4] Suppose that $\vec{V}_1(\vec{x})$ and $\vec{V}_2(\vec{x})$ are two vector fields which have the same divergence and the same curl:

$$\vec{\nabla} \cdot \vec{V}_{1,2}(x) = s(x) \qquad \vec{\nabla} \times \vec{V}_{1,2}(\vec{x}) = \vec{c}(\vec{x})$$

Show that $\vec{V}_1(\vec{x}) = \vec{V}_2(\vec{x})$. [Hint: Use the fact that $\int dV(\vec{\nabla}F)^2 = 0$ for any scalar function $F(\vec{x})$ satisfying Laplace's equation. For extra credit (10 points) prove this.]