Oct. 13, 2005

Physics 600
MIDTERM SOLUTIONS

[1]* — See homework solutions for Set #4

[2] - See class notes pages 18.1 - 18.3

[3] $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$\det(A - \lambda I) = \det \begin{pmatrix} 1-\lambda & 1 \\ 0 & 1-\lambda \end{pmatrix} = 0 \Rightarrow (1-\lambda)^2 = 0 \Rightarrow \lambda = +1$ only solution

$Ax = \lambda x \Rightarrow \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = +1 \begin{pmatrix} a \\ b \end{pmatrix}$

$\Rightarrow \begin{pmatrix} a+b \\ b \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \Rightarrow b = 0 \Rightarrow x = \begin{pmatrix} a \\ 0 \end{pmatrix}$ only solution

Then P has the form

$$P = \left(\begin{pmatrix} a \\ 0 \end{pmatrix} \begin{pmatrix} a \\ 0 \end{pmatrix} \right) = \begin{pmatrix} a & a \\ 0 & 0 \end{pmatrix} \Rightarrow \det P = 0$$

[4] Generally $\det \mathbb{A} = \det \tilde{A}$ $\sim$ = transpose

use:

$$\det A = \sum_{\text{perms}} \left\{ \pm \prod_{i=1}^{n} a_{P(i)\,i} \right\} \qquad P(i) = \text{permutations}$$

For a typical term in $\det A$ $a_{11} = a_{22} = \cdots 0$ if A is antisymm

Then note that

$$\text{in } \det A = \sum \cdots + a_{21} a_{32} a_{13} \cdots$$

$$\text{in } \det \tilde{A} = \sum \cdots \quad a_{12} a_{23} a_{31} \cdots = (-1)^n \det A$$

$$\therefore \det \tilde{A} = (-1)^n \det A \Rightarrow n = \text{odd} \Rightarrow \det A = 0$$