Phys 291
Exam II
March 21, 1996.
Solutions -- Codrington

1. A particle with charge $q = 1.6 \times 10^{-19}$C and mass $m = 1.67 \times 10^{-27}$ Kg is moving with velocity $v = 2 \times 10^6$ m/s through a magnetic field of $B = 1.2$ T. What is the smallest angle between $\vec{v}$ and $\vec{B}$ if the particle experiences a force of $F = 2 \times 10^{-13}$ N?

θ

$$|\vec{F}| = |q\vec{v}\times\vec{B}| = qvB\sin\theta$$

$\boxed{\text{(a) 0.55 radian}}$

$$\theta = \sin^{-1}\frac{F}{qvB} = \sin^{-1}\left(\frac{2\times10^{-13}\ N}{(1.6\times10^{-19}C)(2\times10^{6}\ m/s)(1.2\ T)}\right)$$

(b) 0.67 radian

$$= 31.4^\circ \times \frac{2\pi\ rad}{360^\circ} = .55$$

(c) 1.57 radian

(d) 0.92 radian

(e) none of the above

2. A charged particle with a charge to mass ratio 9.5×10^6 C/Kg is traveling perpendicularly to a uniform magnetic field $B = 2.0$ T. What is the <u>angular</u> frequency of rotation, ω, of this particle?

centripetal accel $a = \frac{v^2}{r}$

ω

$ma = F_B \Rightarrow m\left(\frac{v^2}{r}\right) = qvB \Rightarrow r = \frac{mv}{qB}$

(a) 1.08×10^6 radian/s

period T = time for 1 orbit

$\boxed{\text{(b) } 1.9 \times 10^7 \text{ radian/s}}$

$$= \frac{2\pi r}{v} = \frac{2\pi}{v}\left(\frac{mv}{qB}\right) = \frac{2\pi m}{qB}$$

(c) 2.9×10^8 radian/s

frequency $f = \frac{1}{T} = \frac{qB}{2\pi m}$

(d) 6.0×10^7 radian/s

ang. freq. $\omega = 2\pi f = 2\pi\left(\frac{qB}{2\pi m}\right) = \frac{qB}{m}$

(e) none of the above

$$= \left(\frac{q}{m}\right)B = (9.5\times10^6\ \tfrac{C}{kg})(2\ T)$$

$$= 19\times10^6\ \tfrac{C\cdot T}{kg}$$

3. A coil of 500 turns carries a current of 3.2 A. The plane area of the coil is $|\vec{A}| = 0.2\text{m}^2$. The normal to the plane of the coil makes an angle of $\theta = 37°$ with respect to the $\vec{B}$ field where $B = 1$ T. What is the magnitude of the torque on the coil?

τ

$\vec{\mu} = NiA\hat{n}$

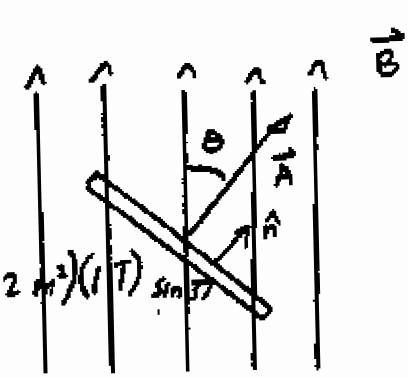
$\boxed{\text{(a) 193.0 Nm}}$

$$|\vec{\tau}| = |\vec{\mu}\times\vec{B}|$$
$$= NiA\,\hat{n}\times\vec{B}$$
$$= NiA\,|\hat{n}||\vec{B}|\sin\theta$$
$$= NiAB\sin\theta$$
$$= (500\ \text{turns})(3.2\ A)(.2\ m^2)(1\ T)\sin 37$$
$$= 192.58$$

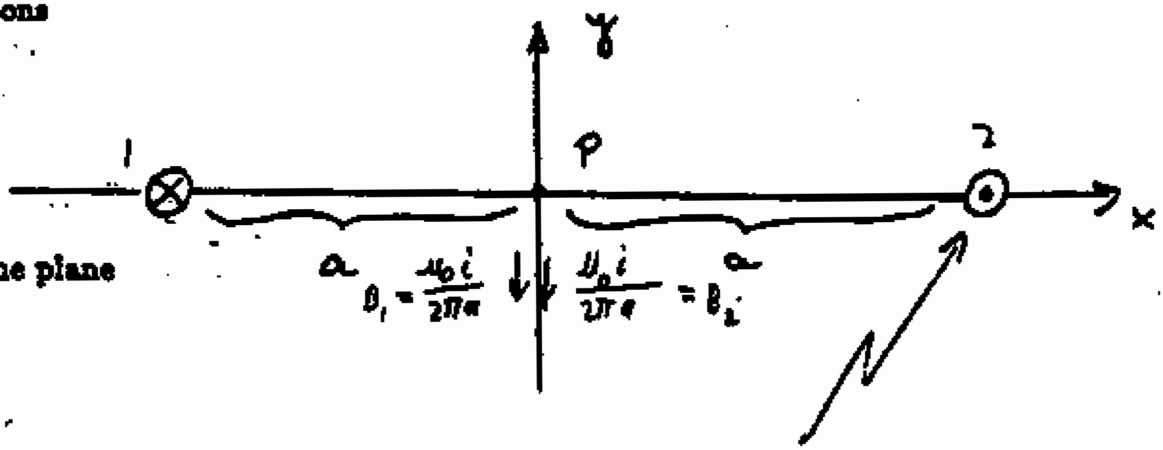
(b) 61.0 Nm

(c) 13.5 Nm

(d) 320 Nm



4. Two infinite straight parallel wires each carry a current of 300 amperes, but in opposite directions



Into the plane

$B_1 = \frac{\mu_0 i}{2\pi a}$ $\downarrow\downarrow$ $\frac{\mu_0 i}{2\pi a} = B_2$

Out of the plane

$a = 0.32$ m. What is the magnitude and direction of the resultant $\vec{B}$ field at the midpoint P?

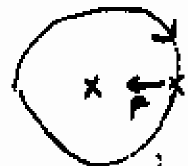
	B	direction
(a)	0 T	
(b)	1.87×10^{-4} T	$+y$
(c)	$\boxed{3.75 \times 10^{-4}\ \text{T}}$	$\boxed{-y}$
(d)	7.5×10^{-4} T	$-y$
(e)	none of the above	

$$B = \frac{2\mu_0 i}{2\pi a} = \frac{\mu_0 i}{\pi a} = \frac{(4\pi\times10^{-7})(300)}{\pi\,(.32)} = 3.75\times10^{-4}\ T$$

in $-y$

5. Two infinite straight parallel wires are 5.6 cm apart. The wires carry currents of 10 A in the same direction. What is the force per unit length between the two wires? Is it attractive or repulsive?

	F/ℓ	
(a)	2.24×10^{-3} N/m	attractive
(b)	2.24×10^{-3} N/m	repulsive
(c)	$\boxed{3.57 \times 10^{-4} \text{ N/m}}$	attractive
(d)	3.57×10^{-4} N/m	repulsive
(e)	none of the above	



length L

$F = i_2 \vec{L} \times \vec{B}$

$= i_2 L B$

$= i_2 L \frac{\mu_0 i_1}{2\pi d}$

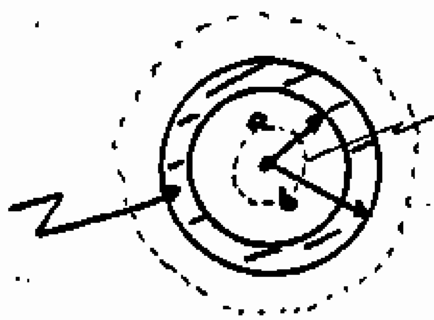
$B = \frac{\mu_0 i_1}{2\pi d}$

$\Rightarrow \frac{F}{L} = \frac{\mu_0 i_1 i_2}{2\pi d} = \frac{(4\pi \times 10^{-7})(10\ A)^2}{2\pi\ (.056\ m)}$

$= 3.57 \times 10^{-4}$

attractive

6. A wire consists of a cylindrical shell of inner radius $a = 1$ cm and an outer radius of $b = 1.1$ cm. The wire carries a total current I of 6.3 A. The current density is uniform across the cross section of the wire.



$\oint \vec{B} \cdot d\vec{l} = 0 \Rightarrow B \cdot 2\pi r = 0 \Rightarrow B = 0$

$\oint \vec{B} \cdot d\vec{l} = \mu_0 i \Rightarrow$

$B = \frac{\mu_0 i}{2\pi \underset{2b}{r}} = \frac{(4\pi \times 10^{-7})(6.3)}{2\pi\ 2(1.1 \times 10^{-2}\ m)} = 5.72 \times 10^{-5}$ T

What is the magnitude of the B field for $r = a/2$ and $r = 2b$

	$B\left(\frac{a}{2}\right)$	$B(2b)$
(a)	$\boxed{0 \text{ T}}$	$\boxed{5.7 \times 10^{-5} \text{ T}}$
(b)	5.7×10^{-5} T	0 T
(c)	3.6×10^{-4} T	0 T
(d)	0 T	3.6×10^{-4} T
(e)	none of the above	

7. An infinite solenoid with a circular cross section and with area 6 cm^2 carries a current of 1500 A. The solenoid has 200 turns per meter along its length. What is the magnitude of the B field in the center of the solenoid?

B

(a) 1.07 T

$B = \mu_0 n i = (4\pi \times 10^{-7})(200 \text{ turns/m})(1500 \text{ A})$

$= .377 \text{ T}$

$\boxed{\text{(b) } 0.38 \text{ T}}$

(c) 0.12 T

(d) 1.71 T

(e) none of the above

8. A plane coil has an Area = 14 cm^2 and consists of 130 turns. A $\vec{B}$ field perpendicular to the plane of the coil has a time dependence ($B(t) = (-2 + 4t + 3t^2)$ T. What is the magnitude of the induced emf $|\mathcal{E}|$ at $t = 0.5$ s?

$\mathcal{E}$

$\Phi = \int_{coil} \vec{B} \cdot dA = BA$

(a) 1.1 V

$\mathcal{E} = -N\frac{d\Phi}{dt} = -NA\frac{dB}{dt} = -NA(4+6t)$

(b) 1.5 V

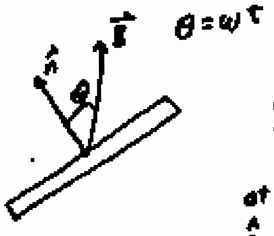
$|\mathcal{E}| = |-NA(4+6t)|\Big|_{t=.5} = (130 \text{ turns})\, 14 \text{ cm}^2 \left(\frac{1 \text{ m}}{100 \text{ cm}}\right)^2$

(c) 0.73 V

$= 1.274 \text{ V}$

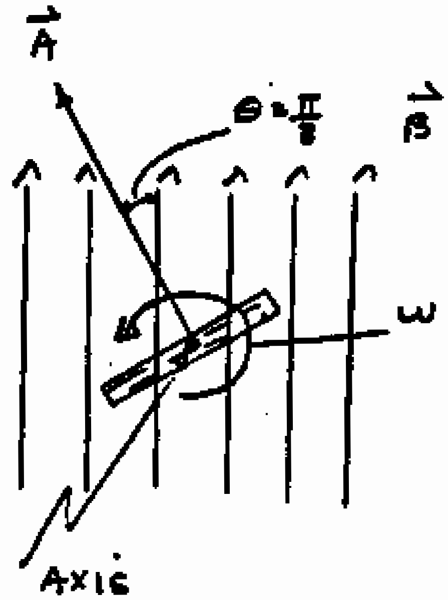
$\boxed{\text{(d) } 1.27 \text{ V}}$

(e) none of the above



at $t=0$, assume $\hat{n}$ is parallel to B

9. A circular coil of 10 turns and area 6 cm² rotates with an angular velocity of $2\pi \cdot 60$ radians per second around an axis perpendicular to the uniform B field. $B = 0.2$ T. The axis of rotation goes through a diameter of the circular cross section of the coil. What is the emf $|\mathcal{E}|$ when $\theta = \frac{\pi}{3}$ as shown above?



$$\Phi = \int_{loop} \vec{B}\cdot d\vec{A} = \int_{loop} B\, dA \cos\omega t$$

$$= B\cos\omega t \left(\int_{loop} dA\right) = BA\cos\omega t$$

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -NBA(-\omega \sin\omega t)$$

$$= NBA\omega \sin\omega t \Big|_{\omega t = \frac{\pi}{3}}$$

$$= (10\text{ turns})(.2\text{ T})\, 6\text{ cm}^2 \times \left(\frac{1\text{m}}{100\text{ cm}}\right)^2 \cdot \left(2\pi \cdot 60\text{ rad/s}\right) \sin(\pi/3)$$

$$= .3917\text{ V}$$

$\mathcal{E}$

(a) 3.9 V

(b) 37.6 V

$\boxed{\text{(c) 0.39 V}}$

(d) 22.5 V

(e) none of the above

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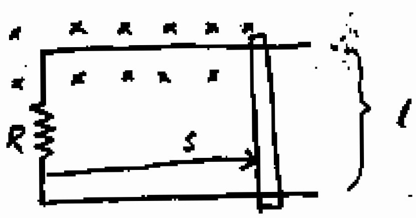
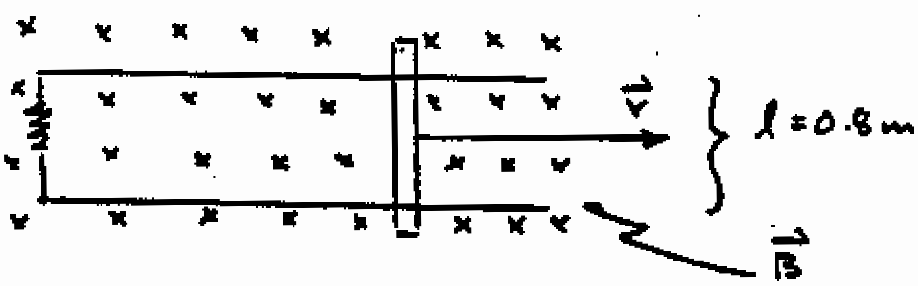
10. Consider a conducting rod traveling on two parallel conducting rails.



$R = 0.1\,\Omega$



$$\Phi = \int \vec{B}\cdot d\vec{A} = -\int B\, dA = -B\int dA = -B\ell s$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\left(B\ell \frac{ds}{dt}\right) = B\ell v$$

$$i = \frac{\mathcal{E}}{R} = \frac{B\ell v}{R}$$

$$P = i^2 R = \left(\frac{B\ell v}{R}\right)^2 R = \frac{B^2\ell^2 v^2}{R}$$

$$B = \frac{\sqrt{PR}}{\ell v} = \frac{\sqrt{(2\text{ W})(.1\,\Omega)}}{(.8\text{ m})(3\text{ m/s})} = .186\text{ T}$$

A uniform B field is perpendicular to the plane of the rails and goes into the plane of the paper. $P = i^2R = 2W$ is dissipated in R when the rail is pulled at constant velocity $v = 3$ m/sec. What is the value of the B field?

B

(a) 0.058 T

(b) 1.42 T

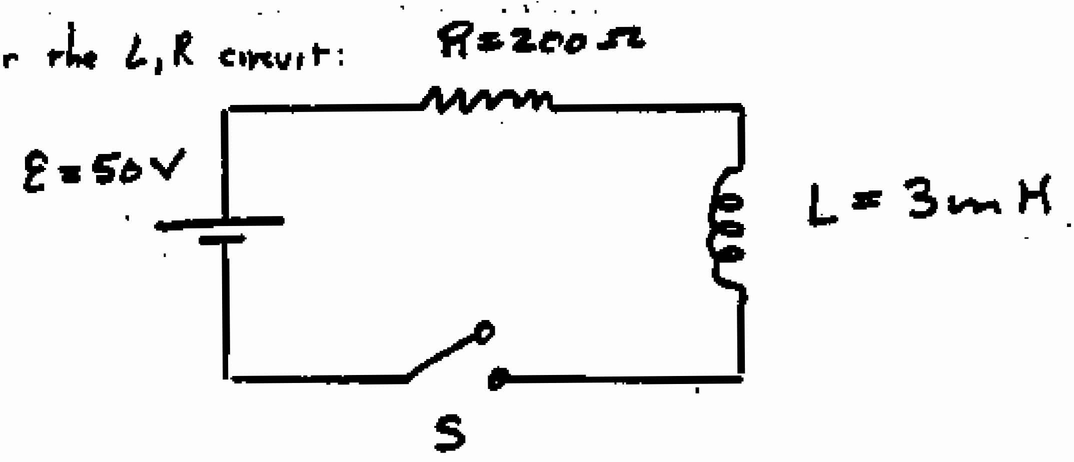
(c) 0.97 T

$\boxed{\text{(d) 0.19 T}}$

(e) none of the above.

Consider the L, R circuit:

11.



at $t = 0$ the switch is closed.

What is $\frac{dI}{dt}$ at $t = 0$ and I at $t = \infty$?

At $t=0$ emf across inductor is $\mathcal{E}_L = 50$ V

$\mathcal{E}_L = L\frac{di}{dt}$

$\Rightarrow \left(\frac{di}{dt}\right)_0 = \frac{\mathcal{E}_L}{L} = \frac{50\ V}{3\times10^{-3}\ H} = 1.66 \times 10^4\ A/s$

$I_\infty = \frac{\mathcal{E}}{R} = \frac{50\ V}{200\ \Omega} = .25\ A$

	$\left(\frac{dI}{dt}\right)_0$	I_∞
(a)	$1.5 \times 10^3 A/s$	0 A
(b)	$0 A/s$	0.25 A
(c)	$1.7 \times 10^4 A/s$	0.25 A
(d)	$1.7 \times 10^4 A/s$	0 A

12. Compute the self inductance per unit length of an infinite solenoid coil. The crossectional area $= 0.8\ m^2$ and $n = 330$ turns per meter. A current $I = 50$ A flows in the coil.

(a) 0.033 H/m

(b) 0.034 H/m

(c) 0.11 H/m

(d) 1.26 H/m

(e) none of the above

(e) none of the above.

$\vec{B} = \mu_0 n i$

$\Phi = BA = \mu_0 n i A$

$N = n\ell$ turns in length ℓ.

Inductance of length ℓ is $L = \frac{N\Phi}{i} = \frac{(n\ell)(\mu_0 n i A)}{i} = n^2\mu_0 \ell A$

Inductance per unit length $= \frac{L}{\ell} = \mu_0 n^2 A$

$= (4\pi\times10^{-7})(330\ \frac{turns}{m})^2(.8\ m^2) = .11$ H/m