Exam II
Phys 241
12/11/97

Solutions - Codrington

A particle (mass = 2.0 mg, charge = −6.0 μC) moves in the positive direction along the x axis with a velocity of 3.0 km/s. It enters a magnetic field B = (2.0i + 3.0j) mT. What is the acceleration of the particle?

(a) (− 27k) m/s²
(b) (+ 27k) m/s²
(c) (+ 18k) m/s²
(d) (− 18k) m/s²
(e) (+ 37k) m/s²

$\vec{F} = q\vec{v} \times \vec{B} = q(3.0\times10^3\ \hat{i}\ \text{m/s}) \times (2\times10^{-3}\ \hat{i} - 3\times10^{-3}\ \hat{j}\ \text{T})$

$= q\, 9\, (\hat{i}\times\hat{j}) = 9q\,\hat{k}$

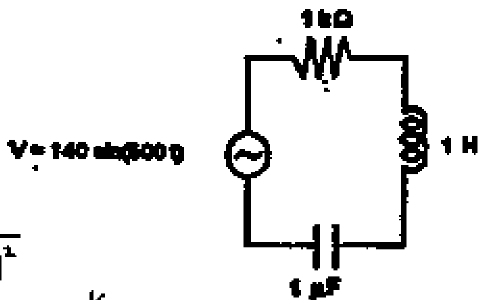
$\vec{F} = m\vec{a} = 9q\,\hat{k}$

$\Rightarrow \vec{a} = \frac{9q\,\hat{k}}{m} = \frac{9(-6\times10^{-6}\ C)}{(2\times10^{-3}\ g)}\,\hat{k}$

$= \frac{9(-6\times10^{-6}\ C)}{(2\times10^{-6}\ kg)}\,\hat{k} = -27\,\hat{k}.$

2. Determine the impedance for the circuit.

(a) 600 Ω
(b) 1200 Ω
(c) 1800 Ω
(d) 2300 Ω
(e) 1100 Ω



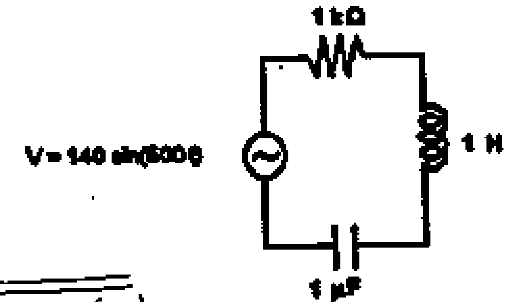

$\omega_d = 500$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega_d L - \frac{1}{\omega_d C}\right)^2}$

$= \left(1000^2 + \left(500\cdot 1 - \frac{1}{500\,(1\times10^{-6}\ F)}\right)^2\right)^{1/2} = 1802.8$

3. Determine the resonant frequency (not the angular frequency) of the circuit.

(a) 159 Hz
(b) 32 Hz
(c) 5 Hz
(d) 500 Hz
(e) 0 Hz




$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1\ H)(1\times10^{-6}\ F)}}$

$= 1000\ \text{rad/s} \times \frac{1\ \text{cycle}}{2\pi\ \text{rad}} = 159.15\ Hz$

4. A long solenoid has a radius of 4.0 cm and has 800 turns/m. The current in the solenoid is 3.0 A. What is the magnitude of the B field at a point 2.2 cm from the axis of the solenoid?

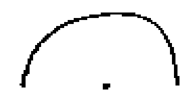
(a) 1 mT
(b) 6 mT
(c) 9 mT
(d) 12 mT
(e) 3 mT

2.2 cm from axis is inside the solenoid.
B field is uniform inside.

$B = \mu_0 n i = (4\pi\times10^{-7}\ \ )(800\ \tfrac{\text{turns}}{m})(3\ A) = 3.02\times10^{-3}\ T$

7. A segment of wire carries a 15 A current and is formed into a semi-circle of radius r = 0.96 m. Determine the magnitude of the magnetic field at the center of the circle along which the wire is placed.

(a) 1.1 μT
(b) 4.9 μT
(c) 1.0 μT
(d) 9.8 μT
(e) 15 μT

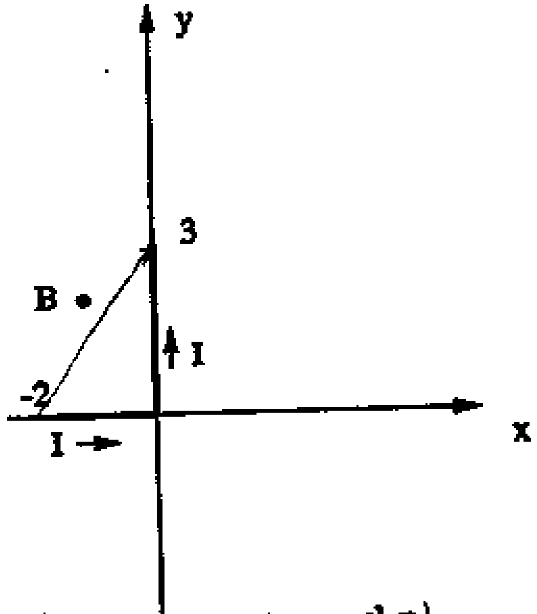


$B = \frac{\mu_0 i \phi}{4\pi r} = \frac{(4\pi\times10^{-7})(15\ A)(\pi)}{4\pi\,(.96)}$

$= 4.91\times10^{-6}\ T$

5. A segment of wire carries a current of 25 A along the $x$ axis from $x = -2.0$ m to $x = 0$ and then along the $y$ axis from $y = 0$ to $y = 3.0$ m. In this region of space, the magnetic field (not due to the wire) is equal to 40 mT in the positive $z$ direction. What is the magnitude of the force on this segment of wire?

(a) 2.0 N
(b) 5.0 N
(c) 1.0 N
(d) 3.6 N
(e) 3.0 N




$\vec{F} = I\vec{L} \times \vec{B} = ILB = (25\ A)\left(\sqrt{(-2)^2 + 3^2}\right)(40 \times 10^{-3}\ T)$

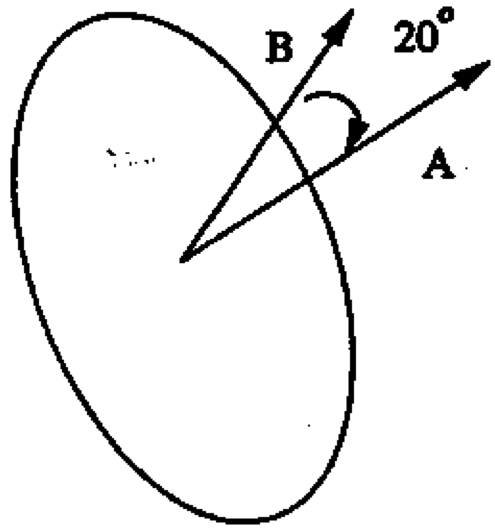
$= 3.61\ N$

6. A current of 4.0 A is maintained in a single circular loop having a circumference of 80 cm. An external magnetic field of 2.0 T is directed such that the angle between the field and the normal to plane of the loop is 20°. Determine the magnitude of the torque exerted on the loop by the magnetic forces acting upon it.

(a) 0.41 N·m
(b) 0.14 N·m
(c) 0.48 N·m
(d) 0.27 N·m
(e) 0.77 N·m

circumference $= 2\pi r = .80\ m$

$\Rightarrow r = \frac{.80}{2\pi}$

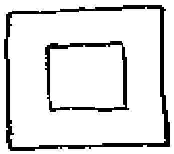



$|\vec{\tau}| = |\vec{\mu} \times \vec{B}| = NiAB \sin\theta = (1\ turn)(4.0\ A)\pi\left(\frac{.80}{2\pi}\right)^2(2\ T)\sin 20°$

$= .139\ N \cdot m$

8. A conducting rod with a square cross section (3.0 cm × 3.0 cm) carries a current of 60 A that is uniformly distributed across the cross section. What is the magnitude of the (line) integral $\oint \mathbf{B} \cdot d\mathbf{s}$ around a square path (1.5 cm × 1.5 cm) if the path is centered on the center of the rod and lies in a plane perpendicular to the axis of the rod?

(a) 14 $\mu$T·m
(b) 75 $\mu$T·m
(c) 19 $\mu$T·m
(d) 57 $\mu$T·m
(e) 38 $\mu$T·m



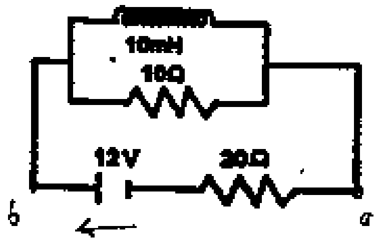
$J = \frac{i}{A} = \frac{60\ A}{(.03\ m)^2}$

$i_{enc} = J\,(.015\ m)^2 = (60\ A)\frac{(.015\ m)^2}{(.03\ m)^2} = 60\ A\left(\frac{1}{2}\right)^2 = 15\ A$

$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc} = (4\pi \times 10^{-7})(15\ A) = 18.8 \times 10^{-6}\ T \cdot m.$

9. For the circuit shown, what is the rate of change of the current in the inductor when the current in the battery is 0.50 A?

(a) 600 A/s
(b) 400 A/s
(c) 200 A/s
(d) 800 A/s
(e) 500 A/s



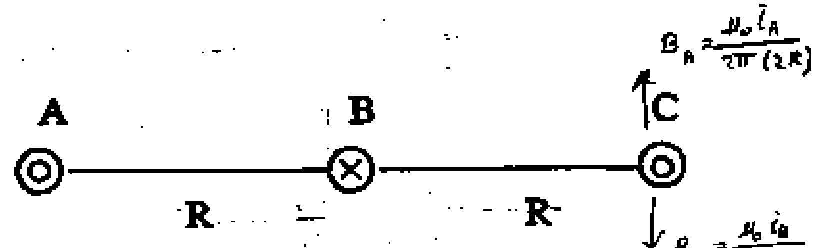

$V_a - V_b = -(20\ \Omega)(.5\ A) + 12 = 2\ V$

$\mathcal{E}_L = 2\ V = -L\frac{di}{dt} \Rightarrow \frac{di}{dt} = -\frac{2}{L} = \frac{2}{(10\times10^{-3}\ H)}$

$= 200\ A/s$

10. The figure shows a cross section of three parallel wires each carrying a current of 15 A. The currents in wires A and C are out of the paper, while that in wire B is into the paper. If the distance $R = 5.0$ mm, what is the magnitude of the force on a 4.0 m length of wire C?

(a) 90 mN
(b) 54 mN
(c) 30 mN
(d) 18 mN
(e) 36 mN




B field down $= \frac{\mu_0 i_B}{2\pi R} - \frac{\mu_0 i_A}{2\pi(2R)} = \frac{\mu_0 i}{2\pi}\left(\frac{1}{R} - \frac{1}{2R}\right) = \frac{\mu_0 i}{4\pi R}$

$= \frac{(4\pi\times10^{-7})(15)}{4\pi(5\times10^{-3}\ m)} = 3\times10^{-4}\ T$

$F = i\vec{L}\times B = iLB = (15)(4\ m)(3\times10^{-4}\ T) = 18\times10^{-3}\ N$

11. A long straight wire (radius = 2.0 mm) carries a current of 40 A. What is the magnitude of the magnetic field inside the wire at 1.5 mm from the axis of the wire?

(a) 3.0 mT
(b) 12 mT
(c) 5.3 mT
(d) 7.4 mT
(e) 8.0 mT

$i = \int \vec{J}\cdot d\vec{A} = JA$

$\Rightarrow J = \frac{i}{A} = \frac{i}{\pi R^2}$

$i_{enc} = J\cdot\pi r^2 = \frac{i}{\pi R^2}\pi r^2 = i\frac{r^2}{R^2}$

$\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{enc}$

$B\cdot 2\pi r = \mu_0 i \frac{r^2}{R^2}$

$\Rightarrow B = \frac{(4\pi\times10^{-7})(40\ A)}{2\pi\ (2\times10^{-3}\ m)^2}(1.5\times10^{-3}\ m) = 3\times10^{-3}\ T$

12. A flat coil of wire consisting of 20 turns, each with an area of 50 cm², is positioned perpendicularly to a uniform magnetic field that increases its magnitude at a constant rate from 2.0 T to 6.0 T in 2.0 s. If the coil has a total resistance of 0.40 Ω, what is the magnitude of the induced current? $\Phi \Rightarrow \int \vec{B}\cdot d\vec{A} = BA$

(a) .70 A
(b) .60 A
(c) .20 A
(d) .80 A
(e) .50 A

emf $\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t} = -\frac{N\Delta B A}{\Delta t}$

$i = \frac{\mathcal{E}}{R} = -\frac{N\Delta B A}{\Delta t\ R} = -\frac{(20\ turns)(6-2\ T)\left(50\ cm^2\times\left(\frac{1\ m}{100\ cm}\right)^2\right)}{(2\ s)(.40\ \Omega)}$

$= .50\ A$

13. A rectangular wire loop (length 60 cm, width 40 cm) lies completely within a perpendicular and uniform magnetic field of magnitude of 0.5 T. If the area of the loop starts decreasing at a rate of $\frac{72\ cm^2}{s}$, what is the magnitude of the induced emf?

(a) 2.8 mV
(b) 5.2 mV
(c) 3.6 mV
(d) 8.4 mV
(e) 10 mV

$\Phi = BA$

$\mathcal{E} = -N\frac{d\Phi}{dt} = -NB\frac{\Delta A}{\Delta t} = -B\frac{\Delta A}{\Delta t}$ (N=1)

$= -(.5\ T)\left(72\ \frac{cm^2}{s}\right)\times\left(\frac{1\ m}{100\ cm}\right)^2 = 3.6\times10^{-3}\ V$