Phys 241 Exam II Fall 1996

Multiple Choice

Solutions – Codrington

1. A charged particle (mass = 4.0 μ g, charge = 5.0 μ C) moves in a region where the only force on it is magnetic. At a point where the speed of the particle is 5.0 km/s, the magnitude of the magnetic field is 8.0 mT, and the angle between the direction of the magnetic field and the velocity of the particle is 60°, what is the magnitude of the acceleration of the particle?

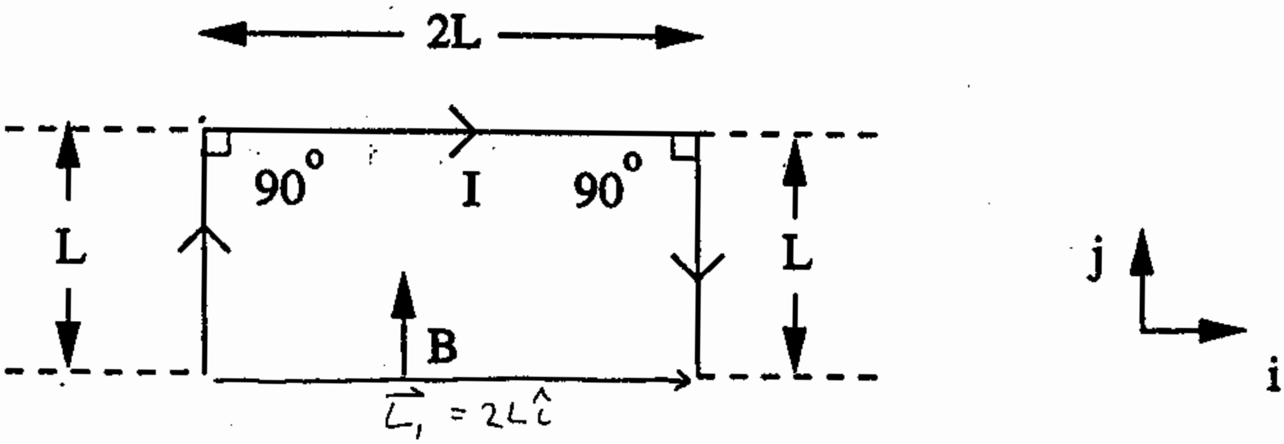
(a) 39 km/s²
$\boxed{\text{(b) 43 km/s}^2}$
(c) 48 km/s²
(d) 52 km/s²
(e) 25 km/s²

$$ma = F_B = q\,\vec{v}\times\vec{B} = q\,v\,B\sin\theta$$
$$a = \frac{q v B \sin\theta}{m} = \frac{(5\times10^{-6}\,C)(5\times10^{3}\,m/s)(8\times10^{-3}\,T)\sin 60^\circ}{4\times10^{-6}\,g\times\frac{1\,kg}{10^3\,g}}$$
$$= 43.30\times10^{3}\ m/s^2$$

2. A straight wire carrying a current I is bent into the shape shown. Determine the net magnetic force on the wire, if the wire is in a uniform magnetic field B directed along the y axis.

(a) 2IBL in the $-z$ direction
$\boxed{\text{(b) 2IBL in the } +z \text{ direction}}$
(c) 4IBL in the $+z$ direction
(d) 4IBL in the $-z$ direction
(e) zero

$$\vec{F} = I\vec{L_1}\times\vec{B}$$
$$= I(2L\hat{i})\times(B\hat{j})$$
$$= 2ILB\,\hat{z}$$



3. What is the kinetic energy of an electron that passes undeviated through perpendicular electric and magnetic fields if E = 4.0 kV/m and B = 8.0 mT?

(a) 0.65 eV
$\boxed{\text{(b) 0.71 eV}}$
(c) 0.84 eV
(d) 0.54 eV
(e) 1.4 eV

$$F = q(\vec{E} + \vec{v}\times\vec{B}) = 0$$
$$\vec{E} = -\vec{v}\times\vec{B} \Rightarrow E = vB \Rightarrow v = \frac{E}{B}$$
$$KE = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(9.1\times10^{-31}\,kg)\left(\frac{4.0\times10^{3}\,V/m}{8\times10^{-3}\,T}\right)^2 =$$
$$= 1.1375\times10^{-19}\,J \times \frac{1\,eV}{1.6\times10^{-19}\,J} = .71\ eV$$

4. **Two straight infinite parallel conductors carry 10 amperes in opposite directions. If the conductors are 25 cm apart, what is the force per unit length on each conductor? It is attractive or repulsive?**

(a) 8×10^{-4} newtons/m - attractive
(b) 8×10^{-4} newtons/m - repulsive
(c) 2×10^{-4} newtons/m - attractive
(d) 2×10^{-4} newtons/m - repulsive
(e) None of the above

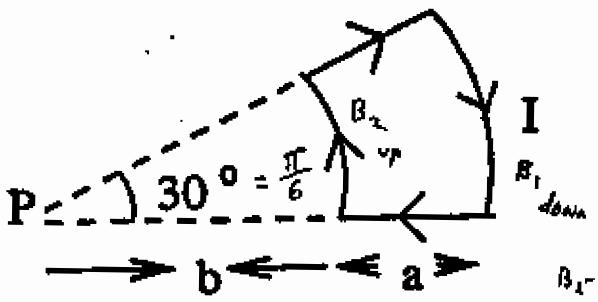
$B = \frac{\mu_0 i_1}{2\pi d}$

$F = i_2 L B = \frac{\mu_0 i_1 i_2 L}{2\pi d}$

$\frac{F}{L} = \frac{\mu_0 i_1 i_2}{2\pi d} = \frac{4\pi \times 10^{-7} \cdot 10 \cdot 10}{2\pi (.25 \text{ m})} = 8 \times 10^{-5} \frac{N}{m}$ repulsive

5. **In the figure shown, if $a = 2.0$ cm, $b = 5.0$ cm, and $I = 20$ A, what is the magnitude of the magnetic field at the point P?**

(a) 4.5 μT
(b) 7.5 μT
(c) 9.0 μT
(d) 6.0 μT
(e) 3.6 μT



$B_1 = \frac{\mu_0 i \frac{\pi}{6}}{4\pi (a+b)}$

$B_2 = \frac{\mu_0 i \frac{\pi}{6}}{4\pi b}$

B field up

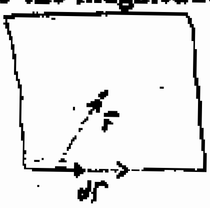
$B_2 - B_1 = \frac{\mu_0 i}{24}\left(\frac{1}{b} - \frac{1}{a+b}\right) = \frac{(4\pi \times 10^{-7})(20\text{ A})}{24}\left(\frac{1}{.05} - \frac{1}{.07}\right)$

$= 5.98 \times 10^{-6}$ T

6. **A straight wire (length = 8.0 m) is bent to form a square. If the wire carries a current of 20 A, what is the magnitude of the magnetic field at the center of the square?**

a = side of square = $a = \frac{8}{4} = 2$ m.

(a) 17 μT
(b) 14 μT
(c) 11 μT
(d) 20 μT
(e) 36 μT



$dB = \frac{\mu_0 i}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2} = \frac{\mu_0 i}{4\pi} \frac{ds \sin\theta}{r^2}$

$\frac{\cos\theta}{\sin\theta} = \frac{s}{a/2} \Rightarrow \left(-\frac{\sin\theta}{\sin\theta} - \frac{\cos^2\theta}{\sin^2\theta}\right) d\theta = \frac{ds}{a/2}$

$\Rightarrow -\frac{1}{\sin^2\theta} d\theta = \frac{ds}{a/2} \Rightarrow ds = -\frac{a/2}{\sin^2\theta} d\theta$

$r \sin\theta = a/2 \Rightarrow r = \frac{a/2}{\sin\theta}$

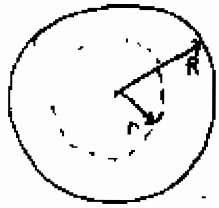
$dB_1 = \frac{\mu_0 i}{4\pi} \frac{\sin\theta}{\left(\frac{a/2}{\sin\theta}\right)^2}\left(-\frac{a/2}{\sin^2\theta} d\theta\right) = -\frac{\mu_0 i}{4\pi} \frac{\sin\theta}{a/2} d\theta \Rightarrow B_1 = \int_{\frac{\pi}{4}}^{\pi/2} -\frac{\mu_0 i}{4\pi} \frac{\sin\theta}{a/2} d\theta$

$B_1 = \frac{2\mu_0 i}{4\pi a}\left[-\cos\theta\right]_{\pi/4}^{\pi/2} = -\frac{2\mu_0 i}{4\pi a} \frac{1}{\sqrt{2}}$

$B = 8B_1 = -\frac{16\mu_0 i}{4\sqrt{2}\pi a} = -\frac{4\mu_0 i}{\sqrt{2}\pi a} = -\frac{4(4\pi \times 10^{-7})(20)}{\sqrt{2}\pi(2)}$

$|B| = \boxed{11.28 \times 10^{-6} \text{ T}}$

7. **A long straight wire (diameter = 2.0 mm) carries a current of 25 A. What is the magnitude of the magnetic field 0.50 mm from axis of the wire?**



(a) 5.0 mT
(b) 10 mT
(c) 0.63 mT
$\boxed{\text{(d) 2.5 mT}}$
(e) 0.01 mT

$i = \int \vec{J}\cdot d\vec{A} = JA \qquad J = \frac{i}{A} = \frac{i}{\pi R^2}$

$i_{enc} = J\cdot\pi r^2 = \frac{i\,\pi r^2}{\pi R^2} = \frac{i r^2}{R^2}$

$\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{enc} \Rightarrow B\cdot 2\pi r = \mu_0\left(\frac{i r^2}{R^2}\right) \Rightarrow B = \left(\frac{\mu_0 i}{2\pi R^2}\right) r$

$B = \frac{(4\pi\times10^{-7}\ T\cdot m/A)(25\ A)(.50\times10^{-3}\ m)}{2\pi\ (1\times10^{-3}\ m)^2} = 2.5\times10^{-3}\ T$

8. **A 40-turn circular coil (radius = 4.0 cm, total resistance = 0.20 Ω) is placed in a uniform magnetic field directed perpendicular to the plane of the coil. The magnitude of the magnetic field varies with time as given by $B = 50\sin(10\pi t)$ mT where t is measured in s. What is the magnitude of the induced current in the coil at 0.10 s?**

(a) 50 mA
$\boxed{\text{(b) 1.6 A}}$
(c) 0.32 A
(d) zero
(e) 0.80 A

$\Phi = \int \vec{B}\cdot d\vec{A} = BA$

$\mathcal{E} = -N\frac{d\Phi}{dt} = -N\frac{dB}{dt}A$

$i = \frac{\mathcal{E}}{R} = -\frac{N}{R}\frac{dB}{dt}A = -\frac{40}{.20\,\Omega}\left\{50\,(10\pi)\cos(10\pi(.1))\times10^{-3}\right\}\pi(.04\,m)^2$

$= -1.571$

9. **A conducting rod (length = 80 cm) rotates at a constant frequency f = 15 Hz about a pivot at one end. A uniform field (B = 60 mT) is directed perpendicularly to the plane of rotation. What is the magnitude of the emf induced between the ends of the rod?**

$\vec{F} = q(\vec{E} + \vec{v}\times\vec{B}) = 0 \Rightarrow -\vec{E} = +\vec{v}\times\vec{B}$

vB

(a) 2.7 V
(b) 2.1 V
(c) 2.4 V
$\boxed{\text{(d) 1.8 V}}$
(e) 3.3 V

$V(R) - V(0) = -\int_0^R \vec{E}\cdot d\vec{r} = -\int_0^R (\vec{v}\times\vec{B})\cdot d\vec{r} = -\int_0^R vB\,dr = -\int_0^R \omega r B\,dr$

$= -\omega B\frac{r^2}{2}\Big|_0^R = -\frac{\omega B R^2}{2} = -\frac{2\pi f B R^2}{2} = -\frac{2\pi(15\,Hz)(60\times10^{-3}\,T)(.8\,m)^2}{2}$

$= 1.809\ V$

10. In the arrangement shown, a conducting bar of negligible resistance slides along horizontal, parallel, frictionless conducting rails connected as shown to a 2.0 Ω resistor. A uniform 1.5 T magnetic field is perpendicular to the plane of the paper. If $L = 60$ cm, at what rate is thermal energy being generated in the resistor at the instant the speed of the bar is equal to 4.2 m/s?

(a) 8.6 W
(b) 7.8 W
(c) 7.1 W
(d) 9.3 W
(e) 1.8 W

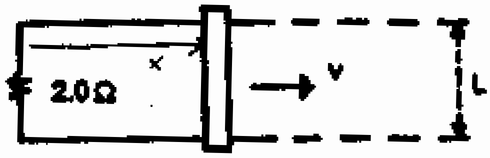
$\Phi = \int \vec{B}\cdot d\vec{A} = BA = B\ell x$

$\mathcal{E} = -N\frac{d\Phi}{dt} = -N B\ell\frac{dx}{dt} = -B\ell v$

$i = \frac{B\ell v}{R}$

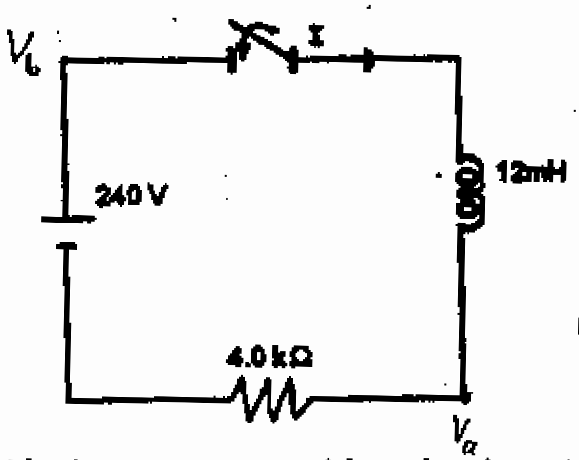
$P = i^2R = \frac{B^2\ell^2v^2}{R} = \frac{(1.5\ T)^2(.6\ m)^2(4.2\ m/s)^2}{2\ \Omega}$

$= 7.14$ W



11. The switch in the figure is closed at $t = 0$ when the current I is zero. When $I = 15$ mA, what is the potential difference across the inductor?

(a) 240 V
(b) 60 V
(c) 0
(d) 180 V
(e) 190 V



$V_a - (4\ k\Omega)(15\ mA) + (240\ V) = V_L$

$V_a - 60 + 240 = V_L$

$V_b - V_a = 180$

12. An ac generator with peak voltage 100 volts is placed across a 10-Ω resistor. What is the average power dissipated?

(a) 100 W
(b) 150 W
(c) 500 W
(d) 1000 W
(e) 2000 W

$V_{rms} = \frac{V_{peak}}{\sqrt{2}}$

$P_{av} = \frac{(V_{rms})^2}{R} = \frac{(V_{peak})^2}{2R} = \frac{(100\ V)^2}{2\cdot 10} = 500$ W

13. An RLC series circuit is to be turned to a resonant frequency $f = 50$ kHz. If the inductance of the circuit is .01 H, what must the capacitance be?

(a) 1 μF
(b) 0.8 μF
(c) 0.08 μF
$\boxed{\text{(d) } 0.001\ \mu\text{F}}$
(e) 0.039 μF

$$\omega = 2\pi f = \frac{1}{\sqrt{LC}} \Rightarrow LC = \frac{1}{(2\pi f)^2}$$

$$\Rightarrow C = \frac{1}{(2\pi f)^2 L} = \frac{1}{(2\pi)^2 (50\times10^3\ Hz)^2 (.01\ H)} = 1.01 \times 10^{-9}\ F$$

14. The voltage 8 sin (400 t) is applied to a series RLC circuit, with $R = 200\ \Omega$, $L = 0.1$ H, and $C = 1\mu$F. What are the impedance Z and the phase angle θ?

$\omega_d = 400$

(a) 200 Ω, −37°
(b) 566 Ω, +87°
$\boxed{\text{(c) } 2468\ \Omega, -85.4°}$
(d) 2540 Ω, −88.8°
(e) 393 Ω, −63°

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega_d L - \frac{1}{\omega_d C}\right)^2}$$

$$= \left(200^2 + \left((400)(.1) - \frac{1}{(400)(1\times10^{-6}\ F)}\right)^2\right)^{1/2} = 2468.1\ \Omega$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{\omega_d L - \frac{1}{\omega_d C}}{R}\right) = \tan^{-1}\left(\frac{(400)(.1) - \frac{1}{(400)(1\times10^{-6}\ F)}}{200}\right)$$

$=$

$$\tan\phi = \frac{X_L - X_C}{R} = -14.81\ \text{rad} \times \frac{360°}{2\pi\ \text{rad}} = -85.35°$$