Physics 241 - Exam II
Fall 2002
Solutions - Codrington

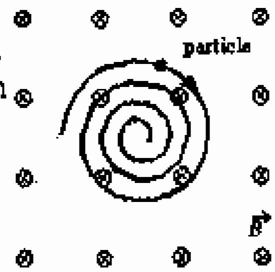
1. An electron travels due north through a vacuum in a region of uniform magnetic field $\vec{B}$ that is also directed due north. It will:
   A) be unaffected by the field
   B) speed up
   C) slow down
   D) follow a right-handed corkscrew path
   E) follow a left-handed corkscrew path

$\vec{v}, \vec{B}$ both point north. Thus

$|\vec{F}_B| = |q\vec{v}\times\vec{B}| = |q||\vec{v}||\vec{B}|\underbrace{\sin 0^\circ}_{0} = 0$ ← angle between $\vec{v},\vec{B}$

magnetic force on electron $= 0 \Rightarrow \boxed{A}$

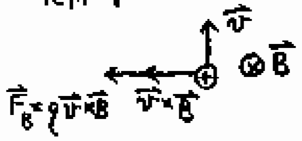
2. A uniform magnetic field is directed into the page. A charged particle, moving in the plane of the page, follows a clockwise spiral of decreasing radius as shown. A reasonable explanation is:
   A) the charge is positive and slowing down
   B) the charge is negative and slowing down
   C) the charge is positive and speeding up
   D) the charge is negative and speeding up
   E) none of the above



radius of orbit is $R = \frac{mv}{|q|B}$

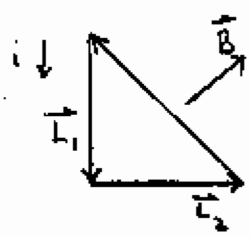
R decreasing $\Rightarrow v$ decreasing, i.e. particle is slowing down.

Assume for a moment that the charge is ⊕. If $\vec{v}$ points upward and $\vec{B}$ points into page, then $\vec{v}\times\vec{B}$ points to the left, so $\vec{F}_B = q\vec{v}\times\vec{B}$ (with $q>0$) points to left:



It follows that the particle would curl to the left, instead of curling to the right as in the diagram above. It follows that the charge must be ⊖, for then $\vec{F}_B = q\vec{v}\times\vec{B}$ will point to the right. Charge is ⊖, $v\downarrow \Rightarrow \boxed{B}$

3. A loop of wire carrying a current of 2.0 A is in the shape of a right triangle with two equal sides, each 15 cm long. A 0.7 T uniform magnetic field is in the plane of the triangle and is perpendicular to the hypotenuse. The resultant magnetic force on the two sides has a magnitude of:
   A) 0
   B) 0.21 N
   C) 0.30 N
   D) 0.41 N
   E) 0.51 N



$\vec{L}_1 + \vec{L}_2 + \vec{L}_3 = 0 \Rightarrow \vec{L}_1 + \vec{L}_2 = -\vec{L}_3$

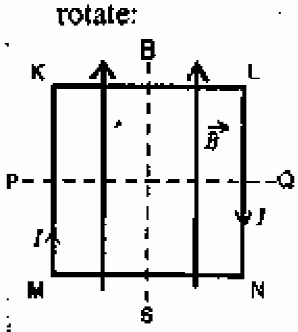
$\vec{F}_1 + \vec{F}_2 = i\vec{L}_1\times\vec{B} + i\vec{L}_2\times\vec{B} = i\underbrace{(\vec{L}_1+\vec{L}_2)}_{-\vec{L}_3}\times\vec{B}$

$= -i\vec{L}_3\times\vec{B}$

$|\vec{F}_1+\vec{F}_2| = i|\vec{L}_3\times\vec{B}| = iL_3B = (2.0\text{ A})\sqrt{(.15)^2+(.15)^2}\,(.7\text{ T})$ ← since $\vec{L}_3 \perp \vec{B}$

$= .30\text{ N} \Rightarrow \boxed{C}$

4. A square loop of wire lies in the plane of the page and carries a current I as shown. There is a uniform magnetic field $\vec{B}$ parallel to the side MK as indicated. The loop will tend to rotate:
   A) about PQ with KL coming out of the page
   B) about PQ with KL going into the page
   C) about RS with MK coming out of the page
   D) about RS with MK going into the page
   E) about an axis perpendicular to the page



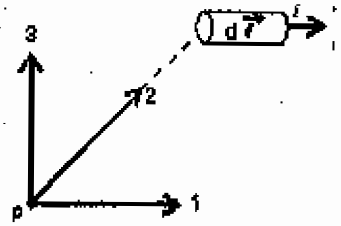
Use right hand rule to find the direction of magnetic dipole moment vector $\vec{\mu}$: curl fingers in direction of current, then thumb points in direction of $\vec{\mu}$ (into page).

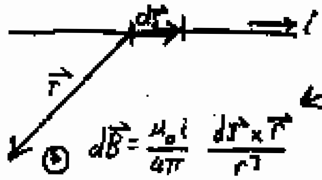
The potential energy $U = -\vec{\mu}\cdot\vec{B} = -|\vec{\mu}||\vec{B}|\cos\theta$ is minimized when $\vec{\mu}$ and $\vec{B}$ point in same direction. Thus $\vec{\mu}$, which points into the page, turns to line up with $\vec{B}$, which points up. Thus the loop rotates about PQ, with the side KL moving out of the page.

Another way to see this is that the torque $\vec{\tau} = \vec{\mu}\times\vec{B}$ points to the right. If we point our thumb in the direction of the torque (to the right), our fingers curl in the direction of the motion. $\Rightarrow \boxed{A}$

5. In the figure, the current element $i d\vec{l}$, the point P, and the three vectors (1, 2, 3) are all in the plane of the page. The direction of $d\vec{B}$, due to this current element, at the point P is:

A) in the direction marked "1"
B) in the direction marked "2"
C) in the direction marked "3"
D) out of the page
E) into the page





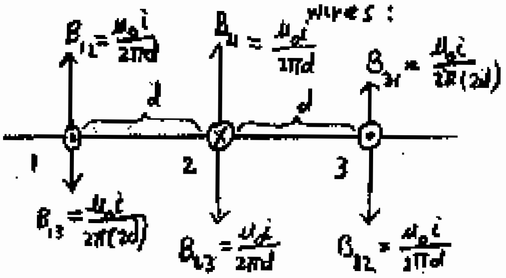
$d\vec{B} = \frac{\mu_0 i}{4\pi} \frac{d\vec{s} \times \vec{r}}{r^3}$ points into page.

Given an element $d\vec{s}$ (or in this case $d\vec{l}$) of a current-carrying wire, we can find the $\vec{B}$-field it produces by using the Biot-Savart law. We find that $d\vec{s} \times \vec{r}$, and therefore $d\vec{B}$, points into the page. $\Rightarrow$ $\boxed{E}$

6. The diagram shows three equally spaced wires that are perpendicular to the page. The currents are all equal, two being out of the page and one being into the page. Rank the wires according to the magnitudes of the magnetic forces on them, from least to greatest.

A) 1, 2, 3
B) 2, 1 and 3 tie
C) 2 and 3 tie, then 1
D) 1 and 3 tie, then 2
E) 3, 2, 1

We first find the $\vec{B}$-field at the position of each wire due to the currents in the other two wires:



Thus the net $\vec{B}$-fields at the positions of wires 1, 2, and 3 satisfy

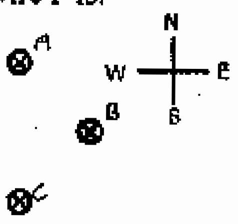
$|\vec{B}_2| = 0$
$|\vec{B}_1| = |\vec{B}_3|$

Since the force on a section $\vec{L}$ of one of the wires is $|\vec{F}| = i|\vec{L} \times \vec{B}| = iLB$, the larger the $\vec{B}$-field at the position of a wire is, the larger the force on that wire. Thus, since $|\vec{F}_2| = 0$, $|\vec{F}_1| = |\vec{F}_3|$, so the ordering from least to greatest is 2, then 1 and 3 tie $\Rightarrow$ $\boxed{B}$.

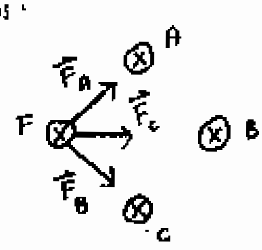
$\vec{L} \perp \vec{B}$

7. Four long straight wires carry equal currents into the page as shown. The magnetic force exerted on wire F is:

A) north
B) east
C) south
D) west
E) zero



"parallel currents attract", so the three force vectors acting on wire F due to the currents in wires A, B, C are as follows:



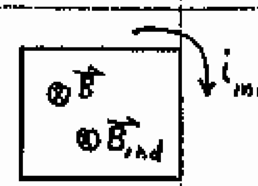
The magnetic force on wire F is the vector sum $\vec{F}_A + \vec{F}_B + \vec{F}_C$. In this vector sum, the vertical components of $\vec{F}_A$, $\vec{F}_C$ will cancel, leaving only an easterly component. $\Rightarrow$ $\boxed{B}$

8. In Ampere's law, $\oint \vec{B} \cdot d\vec{s} = \mu_0 i$, the integration must be over any:

A) surface
B) closed surface
C) path
D) closed path
E) closed path that surrounds all the current producing $\vec{B}$

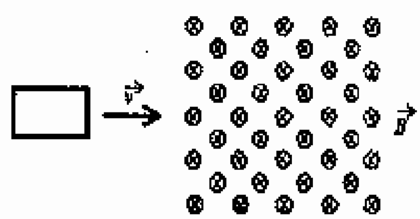
$\int \vec{B} \cdot d\vec{s}$ is a path integral; the circle in $\oint \vec{B} \cdot d\vec{s}$ indicates a closed path $\Rightarrow$ $\boxed{D}$

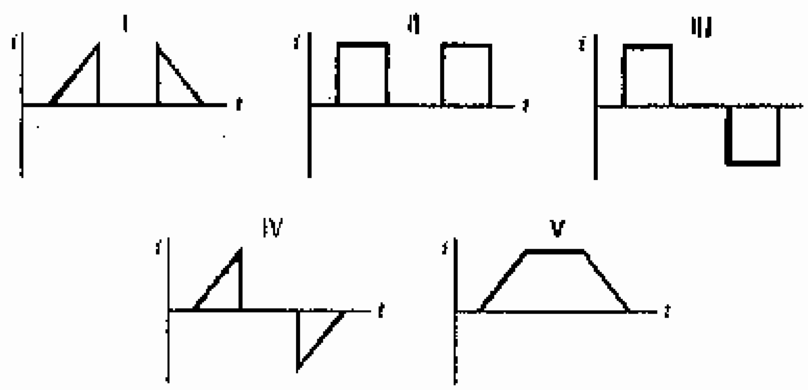
9. A square loop of wire lies in the plane of the page. A decreasing magnetic field is directed into the page. The induced current in the loop is:
   A) counterclockwise
   B) clockwise
   C) zero
   D) depends upon whether or not B is decreasing at a constant rate
   E) clockwise in two of the loop sides and counterclockwise in the other two



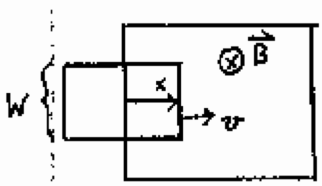
$\vec{B}$ points into page, $|\vec{B}|\downarrow \Rightarrow |\Phi_B|\downarrow$. By Lenz's Law, the induced current flows in a direction to oppose this change. We can oppose this change by reinforcing the $\vec{B}$ field; since the original $\vec{B}$-field pointed into the page, this means that the induced $\vec{B}$-field, $\vec{B}_{ind}$, also points into the page. If we point the thumb of our right hand in the direction of $\vec{B}_{ind}$, our fingers curl in the direction of the induced current (clockwise). $\Rightarrow$ $\boxed{B}$

10. A square loop of wire moves with a constant speed v from a field-free region into a region of uniform B field, as shown. Which of the five graphs correctly shows the induced current i in the loop as a function of time t?
   A) I
   B) II
   C) III
   D) IV
   E) V





Consider a time at which the loop has gone a distance x into the region with a non-zero $\vec{B}$-field, as shown below:

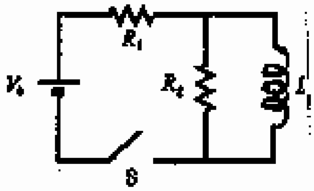


The magnetic flux thru the loop at this time is $\Phi_B = BA = BWx$

The induced emf is $\epsilon = -\frac{d}{dt}(N\Phi_B) = -\frac{d}{dt}(NBWx) = -NBW\underbrace{\frac{dx}{dt}}_{v} = -NBWv$

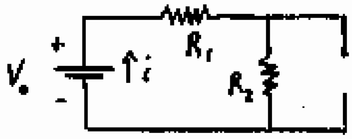
Thus the induced emf doesn't depend on x. It follows that the induced emf (and therefore the induced current) is constant while the loop is entering or leaving the region, and zero while the loop is entirely within the region (since in this case the mag. flux is constant in time). While the loop is entering, $A\uparrow \Rightarrow |\Phi_B|\uparrow$; to oppose this change $\vec{B}_{ind}$ points out of the page, so i is counterclockwise. When the loop is leaving, $A\downarrow$, so $|\Phi_B|\downarrow$; to oppose this change $\vec{B}_{ind}$ points into the page, so i is clockwise.
The only choice consistent with these observations is $\boxed{C}$

11. Immediately after switch S in the circuit shown is closed, the current through the battery shown is:



A) 0
B) V_0/R_1
C) V_0/R_2
D) $V_0/(R_1 + R_2)$
E) $V_0(R_1 + R_2)/(R_1 R_2)$

Before the switch is closed, the current thru the inductor is zero. Since the inductor doesn't allow the current thru it to change instantaneously, this means that the current thru the inductor will also be zero immediately after the switch is closed. It follows that we can replace the inductor with an open circuit:



loop equation: $V_0 - iR_1 - iR_2 = 0 \Rightarrow i = \dfrac{V_0}{R_1+R_2} \Rightarrow \boxed{D}$

12. An 6.0-mH inductor and a 3.0-Ω resistor are wired in series to a 12-V ideal battery. A switch in the circuit is closed at time 0, at which time the current is zero. 2.0 ms later the energy stored in the inductor is:

A) 0
B) 2.5×10^{-2} J
C) 1.9×10^{-2} J
D) 3.8×10^{-2} J
E) 9.6×10^{-3} J

Connecting battery ⇒ use rise of current equation

$i(t) = \underbrace{i_f}_{\mathcal{E}/R}\,(1 - e^{-t/\tau_L})$ where $t = 2.0\times10^{-3}$ s,

$\tau_L = \dfrac{L}{R} = \dfrac{6.0\times10^{-3}\ H}{3.0\ \Omega} = 2\times10^{-3}$ s.

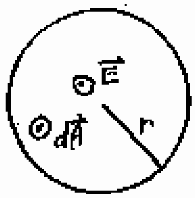
$$U = \tfrac{1}{2}Li^2 = \tfrac{1}{2}L\left(\tfrac{\mathcal{E}}{R}\left(1 - e^{-\frac{2.0\ ms}{2.0\ ms}}\right)\right)^2 = \tfrac{1}{2}(6.0\times10^{-3}\ H)\left(\tfrac{12\ V}{3.0\ \Omega}(1-e^{-1})\right)^2 = 1.9\times10^{-2}\ J \Rightarrow \boxed{C}$$

13. A 1.2-m radius cylindrical region contains a uniform electric field that is perpendicular to the cross sections of the region. At $t = 0$ the field is 0 and increases uniformly to 200 V/m at $t = 5.0$ s. The total displacement current through a cross section of the region is:

A) 4.5×10^{-16} A
B) 2.0×10^{-15} A
C) 3.5×10^{-10} A
D) 1.6×10^{-9} A
E) 8.0×10^{-9} A

Electric flux $\Phi_E = \int \vec{E}\cdot d\vec{A} = E\underbrace{A}_{\pi r^2}$

Displacement current $i_d = \epsilon_0 \dfrac{d\Phi_E}{dt} = \epsilon_0 \dfrac{d}{dt}(EA)$



$$i_d = \epsilon_0 A \frac{dE}{dt} \approx \epsilon_0 \pi r^2 \frac{\Delta E}{\Delta t} = \left(8.85\times10^{-12}\ \tfrac{T\cdot m}{A}\right)\pi(1.2\ m)^2\ \frac{200\ \frac{V}{m} - 0}{5.0\ s - 0} = 1.60\times10^{-9}\ A \Rightarrow \boxed{D}$$

14. At time $t = 0$ the charge on the 50-μF capacitor in an LC circuit is 15 μC and there is no current. If the inductance is 20 mH the maximum current is:

A) 15 nA
B) 15 μA
C) 6.7 mA
D) 15 mA
E) 15 A

In an LC circuit, there's no resistor to dissipate energy

⇒ energy in LC circuit stays constant with time

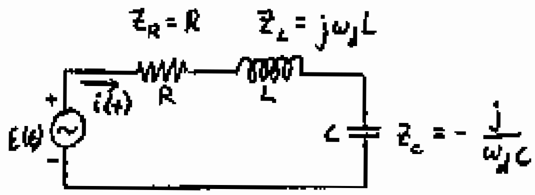
⇒ (energy at a time when all energy is stored in the inductor) = (energy at a time when all energy is stored in capacitor)

⇒ $\tfrac{1}{2}Li_{max}^2 = \tfrac{1}{2C}q_{max}^2$

⇒ $i_{max} = \dfrac{1}{\sqrt{LC}}\,q_{max} = \dfrac{1}{\sqrt{(20\times10^{-3}\ H)(50\times10^{-6}\ F)}}(15\ \mu C) = 15\ mA \Rightarrow \boxed{D}$

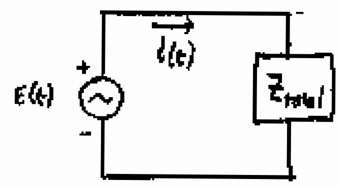
15. An ac generator producing 10 V (rms) at 200 rad/s is connected in series with a 50-Ω resistor, a 400-mH inductor, and a 200-μF capacitor. The rms voltage (in volts) across the inductor is:

A) 2.5
B) 3.4
C) 6.7
D) 10.0
E) 10.8



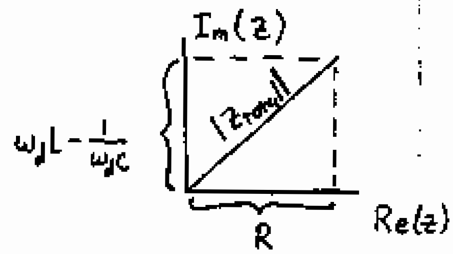
$E(t) = \mathcal{E}_m \sin(\omega_d t)$
← driving freq $\omega_d = 200$ rad/s

Impedances in series add (like resistors): Combine R, L, and C into one impedance Z_{total}



$$Z_{total} = Z_R + Z_L + Z_c = R + j\omega_d L - j\frac{1}{\omega_d C} = R + j\left(\omega_d L - \frac{1}{\omega_d C}\right)$$

Amplitude of $i(t)$ = $\dfrac{\text{Amplitude of } E(t)}{|Z_{total}|}$ (like $V = \frac{I}{R}$)



$$\Rightarrow I_m = \frac{\mathcal{E}_m}{\sqrt{R^2 + \left(\omega_d L - \frac{1}{\omega_d C}\right)^2}}$$

$i(t)$ is the current thru the inductor, so

$$\underbrace{\text{Amplitude of voltage across inductor}}_{V_L} = (\text{Amplitude of } i(t))\,|Z_L|$$

$$= I_m \underbrace{|j\omega_d L|}_{\omega_d L} = I_m \omega_d L$$

$$\Rightarrow V_L = I_m \omega_d L = \left(\frac{\mathcal{E}_m}{\sqrt{R^2 + \left(\omega_d L - \frac{1}{\omega_d C}\right)^2}}\right)\omega_d L = \frac{\mathcal{E}_m \omega_d L}{\sqrt{R^2 + \left(\omega_d L - \frac{1}{\omega_d C}\right)^2}}$$

$$\Rightarrow \underbrace{\frac{V_L}{\sqrt{2}}}_{V_{L,rms}} = \frac{(\mathcal{E}_m/\sqrt{2})\,\omega_d L}{\sqrt{R^2 + \left(\omega_d L - \frac{1}{\omega_d C}\right)^2}} = \frac{(10 \text{ V rms})(200 \text{ rad/s})(400\times10^{-3} \text{ H})}{\sqrt{(50\,\Omega)^2 + \left((200 \text{ rad/s})(400\times10^{-3}\text{ H}) - \frac{1}{(200 \text{ rad/s})(200\times10^{-6}\text{ F})}\right)^2}}$$

$= 10.80$ V ⇒ $\boxed{E}$

16. The primary of an ideal transformer has 100 turns and the secondary has 600 turns. Then:

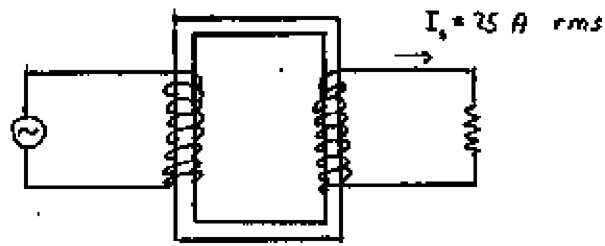
A) the power in the primary circuit is less than that in the secondary circuit
B) the currents in the two circuits are the same
C) the voltages in the two circuits are the same
D) the primary current is six times the secondary current
E) the frequency in the secondary circuit is six times that in the primary circuit

$$I_s = I_p \frac{N_p}{N_s} \Rightarrow I_p = I_s \frac{N_s}{N_p} = I_s \frac{600}{100} = 6 I_s \Rightarrow \boxed{D}$$

secondary primary

17. In an ideal 1:8 step-down transformer, the primary power is 10 kW and the secondary current is 25 A. The primary voltage is:

A) 25,600 V
B) 3200 V
C) 400 V
D) 50 V
E) 6.25 V



Step-down $\Rightarrow V_s < V_p \Rightarrow \frac{V_s}{V_p} = \frac{1}{8} \Rightarrow \frac{N_s}{N_p} = \frac{V_s}{V_p} = \frac{1}{8}$

$I_s = I_p \frac{N_p}{N_s} \Rightarrow I_p = I_s \frac{N_s}{N_p} = (25 \text{ A rms})\left(\frac{1}{8}\right) = \frac{25}{8} \text{ A rms}$

$V_p = \frac{\text{Primary power}}{I_p} = \frac{10{,}000 \text{ W}}{\frac{25}{8} \text{ A rms}} = 3200 \text{ V rms} \Rightarrow \boxed{B}$