Physics 241 Exam II
Nov. 6, 2001

Solutions - Codrington

1) A point charge, q, traveling at velocity, v, enters a region with uniform magnetic field B=5i+10k Tesla. There are three possibilities for the velocity: v_1=20i m/s, v_2=15j m/s, and v_3=30 k m/s, resulting in three possible forces with respective magnitudes F_1, F_2, and F_3. Which of the following statements is correct? [10 points.]

(1) $F_1<F_2<F_3$
(2) $F_2<F_1<F_3$
(3) $F_2<F_1<F_3$
(4) $F_1=F_2<F_3$
(5) $F_1<F_3<F_2$
(6) $F_3<F_1=F_2$
(7) $F_2=F_1<F_3$
(8) $F_3<F_1<F_2$
(9) $F_3<F_2=F_1$
(10) $F_3<F_2<F_1$

$$|\vec{F}| = |q\vec{v}\times\vec{B}| = |q||\vec{v}\times\vec{B}|$$

$$F_1 = |\vec{F}_1| = |q||\vec{v}_1\times\vec{B}| = |q||(20\hat{i})\times(5\hat{i}+10\hat{k})|$$
$$= |q||100\underbrace{\hat{i}\times\hat{i}}_{0} + 200\underbrace{\hat{i}\times\hat{k}}_{-\hat{j}}| = |q||-200\hat{j}| = 200|q|$$

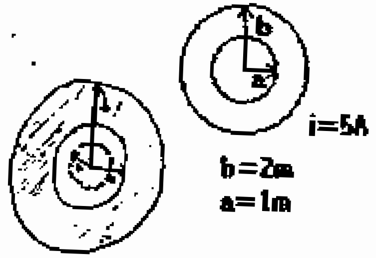
$$F_2 = |\vec{F}_2| = |q||\vec{v}_2\times\vec{B}| = |q||(15\hat{j})\times(5\hat{i}+10\hat{k})|$$
$$= |q||75\underbrace{\hat{j}\times\hat{i}}_{-\hat{k}} + 150\underbrace{\hat{j}\times\hat{k}}_{\hat{i}}| = |q|\sqrt{150^2+75^2} = 167.7|q|$$

$$F_3 = |\vec{F}_3| = |q||\vec{v}_3\times\vec{B}| = |q||(30\hat{k})\times(5\hat{i}+10\hat{k})|$$
$$= |q||150\underbrace{\hat{k}\times\hat{i}}_{\hat{j}} + 300\underbrace{\hat{k}\times\hat{k}}_{0}| = 150|q|$$

$$\therefore \quad F_3 < F_2 < F_1$$

$$\Rightarrow \boxed{(10)}$$

2) An infinitely long, cylindrical conductor has a hollow core. The outer radius is b=2m, while the hollow core radius is a=1m. A current of 5A flows through the conductor. What is the intensity of the magnetic field at a position of 0.8m from the central axis of the cylindrical conductor? [5 points.]



(1) 200 nT
(2) 160 mT
(3) 213 mT
(4) 267 nT
(5) 1 T
(6) 0 T
(7) 0.2 T
(8) 1.25 μT
(9) 0.25 μT
(10) 318 pT

Amperes law:
$$\oint_{\text{loop}} \vec{B}\cdot d\vec{s} = \mu_0 \underbrace{i_{\text{enc}}}_{0}$$
$$\underbrace{\oint |\vec{B}| ds}_{|\vec{B}(r)| \boxed{\oint_{\text{loop}} ds}_{2\pi r}}$$

$$\therefore |\vec{B}(r)| 2\pi r = 0$$
$$\Rightarrow |\vec{B}(r)| = 0$$
$$\Rightarrow \boxed{(6)}$$

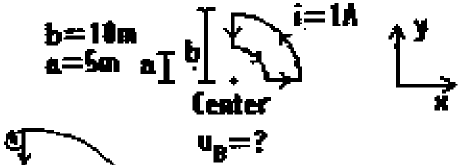
3) Part II of Problem 2. What is the intensity of the magnetic field at a distance of 5m from the central axis? [5 points.]

(1) 200 nT
(2) 160 mT
(3) 213 mT
(4) 267 nT
(5) 1 T
(6) 0 T
(7) 0.2 T
(8) 1.25 μT
(9) 0.25 μT
(10) 318 pT

Amperes Law:
$$\oint_{\text{loop}} \vec{B}\cdot d\vec{s} = \mu_0 \underbrace{i_{\text{enc}}}_{i}$$
$$\underbrace{\oint |\vec{B}| ds}_{|\vec{B}(r)| \boxed{\oint_{\text{loop}} ds}_{2\pi r}}$$

$$\Rightarrow |\vec{B}(r)| = \frac{\mu_0 i}{2\pi r} = \frac{(4\pi\times10^{-7}\ \frac{T\,m}{A})(5\ A)}{2\pi(5\ m)} = 2\times10^{-7}\ T = 200\ nT$$
$$\Rightarrow \boxed{(1)}$$

7) A wire consisting of two arc segments of circles with radii, a=5m and b=10m and two straight segments resides in the x-y plane as shown in figure. A current of 1A circulates counter clockwise. Given that the arcs subtend an angle of 90° what is the magnetic field energy density at the center point of the arcs (circles)? [10 points.]



(1) 3.2 nJ/m³

(2) 1.3 nJ/m³

(3) 98 pJ/m³

(4) 53 pJ/m³

(5) 7.7 mJ/m³

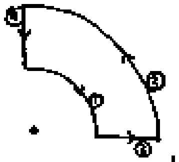
(6) 5.1 mJ/m³

(7) 882 pJ/m³

(8) 46 pJ/m³

(9) 0.7 pJ/m³

(10) 2.8 pJ/m³



Side ①:

$d\vec{B}_1 = \frac{\mu_0 i}{4\pi}\frac{d\vec{s}\times\hat{r}}{r^2}$ points into page by right hand rule.

$|\vec{B}_1| = \frac{\mu_0 i \phi}{4\pi a} = \frac{(4\pi\times10^{-7}\ \frac{T\cdot m}{A})(1\,A)(\frac{\pi}{2})}{4\pi(5\,m)} = 3.1416\times10^{-4}$

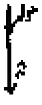
Side ②

$d\vec{B}_2 = \frac{\mu_0 i}{4\pi}\frac{d\vec{s}\times\hat{r}}{r^2} = 0$

since $|d\vec{s}\times\hat{r}| = |d\vec{s}||\hat{r}|\overbrace{\sin(180°)}^{0} = 0$

$\therefore \vec{B}_2 = 0$

Side ④



$|d\vec{s}\times\hat{r}| = |d\vec{s}||\hat{r}|\overbrace{\sin(0°)}^{0} = 0$

$\Rightarrow d\vec{B}_4 = \frac{\mu_0 i}{4\pi}\frac{d\vec{s}\times\hat{r}}{r^2} = 0$

$\therefore \vec{B}_4 = 0$

Side ③

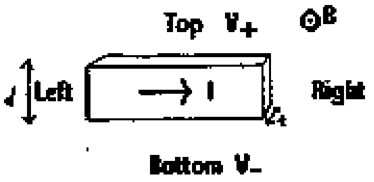
$|\vec{B}_3| = \frac{\mu_0 i \phi}{4\pi b} = \frac{(4\pi\times10^{-7}\ \frac{T\cdot m}{A})(1\,A)(\frac{\pi}{2})}{4\pi(10\,m)} = 1.5708\times10^{-8}$

$d\vec{B}_3 = \frac{\mu_0 i}{4\pi}\frac{d\vec{s}\times\hat{r}}{r^2}$ points out of page

$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4 = 3.1416\times10^{-8} - 1.5708\times10^{-8} = 1.5708\times10^{-8}$ (into page)

Energy density: $u_B = \frac{|\vec{B}|^2}{2\mu_0} = \frac{(1.5708\times10^{-8}\,T)^2}{2(4\pi\times10^{-7}\ \frac{T\cdot m}{A})} = 98.17\times10^{-12}\,J = 98.2\ pJ \Rightarrow \boxed{(3)}$

8) A rectangular conductor of uniform cross section (width, w=5cm, and thickness, t=1cm) carries a current, I=2.5A, from left to right. A Hall voltage, V=10mV, develops across the conductor with the positive potential towards the top with the application of a magnetic field, B=1T, out of the page. Given that the current carries are of charge, e=1.6E-19C, in magnitude without specification of sign, what are the density of the carriers and the sign of the carrier charge? [10 points.]



(1) 1.56E23 /m³ and "positive"

(2) 3.12E22 /m³ and "positive"

(3) 1.56E23 /m³ and "negative"

(4) 3.12E22 /m³ and "negative"

(5) 2.23E23 /m³ and "positive"

(6) 3.78E23 /m³ and "positive"

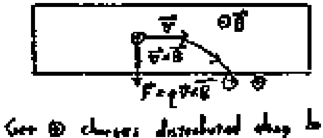
(7) 2.23E23 /m³ and "negative"

(8) 0.63E22 /m³ and "negative"

(9) 0.63E22 /m³ and "positive"

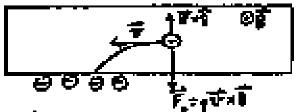
(10) None of the above

If charge carriers were ⊕:



Get ⊕ charges distributed along bottom, ⊖ along top, ⇒ $\vec{E}$-field points up ⇒ bottom at higher potential (Wrong! given that top is at higher potential)

If charge carriers were ⊖:

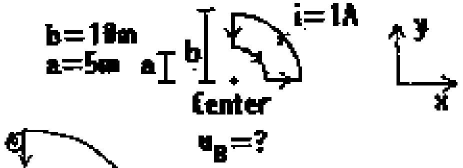


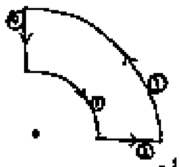
Get ⊖ charges distributed along bottom, ⊕ along top, ⇒ $\vec{E}$-field points down ⇒ top at higher potential. Thus the charge carriers are "negative"

Number density of charge carriers $= n = \frac{Bi}{V(\frac{A}{d})e} = \frac{(1\,T)(2.5\,A)}{(10\times10^{-3}\,V)\frac{(5\times10^{-2}\,m)}{(1\times10^{-2}\,m)}(1.6\times10^{-19}\,C)}$

$= 1.56\times10^{23}\ \frac{1}{m^3} \Rightarrow \boxed{(3)}$

7) A wire consisting of two arc segments of circles with radii, a=5m and b=10m and two straight segments resides in the x-y plane as shown in figure. A current of 1A circulates counter clockwise. Given that the arcs subtend an angle of 90° what is the magnetic field energy density at the center point of the arcs (circles)? [10 points.]





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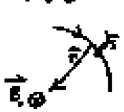
Side ①



$d\vec{B}_1 = \frac{\mu_0 i}{4\pi}\frac{d\vec{s}\times\hat{r}}{r^2}$ points into page by right hand rule

$|\vec{B}_1| = \frac{\mu_0 i \theta}{4\pi a} = \frac{(4\pi\times10^{-7}\ \frac{T\cdot m}{A})(1A)(\frac{\pi}{2})}{4\pi(5m)} = 3.1416\times10^{-8}$

Side ②: $d\vec{B}_2 = \frac{\mu_0 i}{4\pi}\frac{d\vec{s}\times\hat{r}}{r^2} = 0$ since $|d\vec{s}\times\hat{r}| = |d\vec{s}||\hat{r}|\overset{0}{\sin(180°)} = 0$ $\therefore \vec{B}_2 = 0$

Side ④: $|d\vec{s}\times\hat{r}| = |d\vec{s}||\hat{r}|\overset{0}{\sin(0°)} = 0$ $\Rightarrow d\vec{B}_4 = \frac{\mu_0 i}{4\pi}\frac{d\vec{s}\times\hat{r}}{r^2} = 0$ $\therefore \vec{B}_4 = 0$

Side ③

$|\vec{B}_3| = \frac{\mu_0 i \theta}{4\pi b} = \frac{(4\pi\times10^{-7}\ \frac{T\cdot m}{A})(1A)(\frac{\pi}{2})}{4\pi(10m)} = 1.5708\times10^{-8}$

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Energy density: $u_B = \frac{|\vec{B}|^2}{2\mu_0} = \frac{(1.5708\times10^{-8}\ T)^2}{2(4\pi\times10^{-7}\ \frac{T\cdot m}{A})} = 98.17\times10^{-12}\ J = 98.2\ pJ \Rightarrow \boxed{(3)}$

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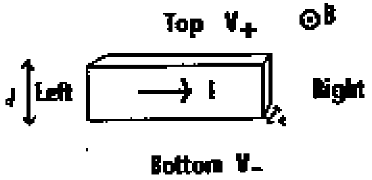

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If charge carriers were ⊕:

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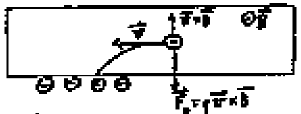

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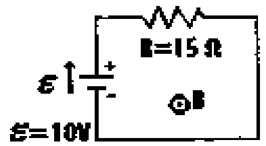
Thus the charge carriers are "negative"

number density of charge carriers = $n = \frac{Bi}{V(\frac{A}{d})e} = \frac{(1\ T)(2.5\ A)}{(10\times10^{-3}\ V)\frac{(5\times10^{-2}\ m)^2}{(1\times10^{-2}\ m)}(1.6\times10^{-19}\ C)}$

$= 1.56\times10^{23}\ \frac{1}{m^3} \Rightarrow \boxed{(3)}$

9) The circuit containing the rectangular loop (2m x 3m) is placed in a uniform, but time varying magnetic field out of the page, B=2.5T− (3.8T/s) t . What is the current in the circuit? [10 points.]



(1) 2.19 A CCW

(2) 2.19 A CW

(3) 1.52 A CCW

(4) 1.52 A CW

(5) 0 A.

(6) 0.85 A CCW

(7) 0.85 A CW

(8) 1.25 A CCW

(9) 1.25 A CW

(10) 0.67 A CW

Current due to battery is $i = \frac{\varepsilon}{R} = \frac{10\ V}{15\ \Omega} = .666\ A$ CW (clockwise)

Mag flux $\Phi_B = B(t)A$

Induced emf $\varepsilon = -\frac{d}{dt}(N\overset{N=1}{\Phi_B}) = -\frac{d}{dt}(B(t)A) = -A\frac{d}{dt}(2.5 - 3.8t)$

$= 3.8A = (3.8\ T/s)(2 \cdot 3\ m^2) = 22.8\ \frac{T \cdot m^2}{s}$

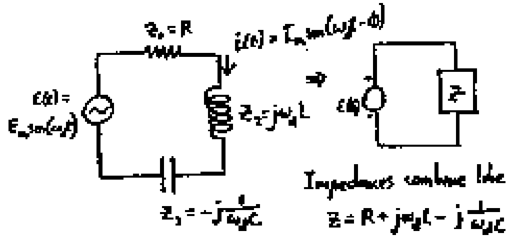
Induced current $i_{ind} = \frac{\varepsilon}{R} = \frac{22.8}{15} = 1.52\ A$

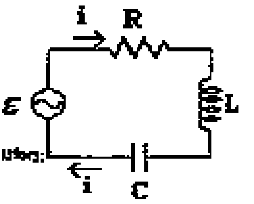
$\vec{B}$ field points out of page and is decreasing with time ⇒ flux decreases with time. By Lenz's Law, oppose change by making induced $\vec{B}$-field point out of page, so by right hand rule induced current flows CCW (counter-clockwise)

∴ Current is $1.52 - .666 = .853\ A$ CCW

⇒ $\boxed{(6)}$

10) An RLC circuit is driven by an alternating voltage with E=E$_m$sin ω$_d$t, E$_m$=20V. Here R=100Ω, L=100µH, and C=40nF. What is the RMS current when driving at resonance? [5 points.]





Impedances combine like resistors:

$Z = R + j\omega_d L - j\frac{1}{\omega_d C}$

$= R + j(\omega_d L - \frac{1}{\omega_d C})$

(1) 0.20 A
(2) 0.40 A
(3) 0.10 A
(4) 0.65 A
(5) 0.14 A
(6) 1.13 A
(7) 0.06 A
(8) 0.55 A
(9) 0.78 A
(10) 0.8 A

Amplitude of current waveform $I_m = \frac{E_m}{|Z|} = \frac{E_m}{\sqrt{R^2 + (\omega_d L - \frac{1}{\omega_d C})^2}}$

Resonant freq is $\omega_0 = \frac{1}{\sqrt{LC}}$. Circuit driven at resonance is $\omega_d = \omega_0 = \frac{1}{\sqrt{LC}}$

RMS Current

$I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{E_m}{\sqrt{2}\sqrt{R^2 + (\omega_d L - \frac{1}{\omega_d C})^2}} = \frac{E_m}{\sqrt{2}\sqrt{R^2 + \underbrace{(\frac{1}{\sqrt{LC}}L - \frac{1}{\frac{1}{\sqrt{LC}}C})^2}_{\sqrt{\frac{L}{C}} - \sqrt{\frac{L}{C}} = 0}}}$

$= \frac{E_m}{\sqrt{2}\sqrt{R^2}} = \frac{E_m}{\sqrt{2}R} = \frac{20\ V}{\sqrt{2}\ 100\ \Omega} = .141\ A$

⇒ $\boxed{(5)}$

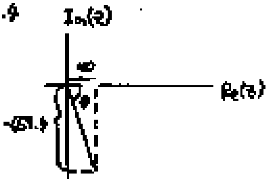
11) If driving at a frequency of 6000 Hz (f=6000/s), what is the phase shift, φ, of the current, I? Note the convention: I=I sin (ω$_d$t − φ)[5 points.]

$f_d = 6000\ Hz \qquad \omega_d = 2\pi f_d = 2\pi \cdot 6000\ \frac{rad}{s}$

(1) 37.8 degrees
(2) −81.4 degrees
(3) −37.8 degrees
(4) −51.7 degrees
(5) 77.6 degrees
(6) −88.6 degrees
(7) 12.3 degrees
(8) 88.6 degrees
(9) −77.6 degrees
(10) 82.0 degrees

$Z = R + j(\omega_d L - \frac{1}{\omega_d C}) = 100 + j\left((2\pi \cdot 6000)(100 \times 10^{-6} H) - \frac{1}{(2\pi \cdot 6000)(40 \times 10^{-9} F)}\right)$

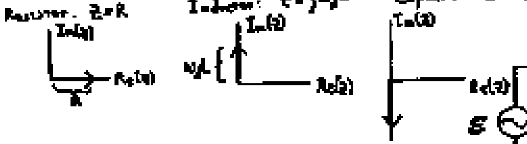
$= 100 - j659.4$

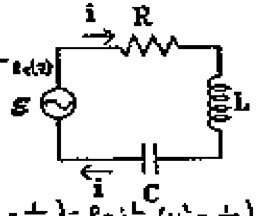


$\phi = \tan^{-1}\left(\frac{-659.4}{100}\right) = -81.37°$ ⇒ $\boxed{(2)}$

12) For the circuit of series RLC circuit of Problem 11, which of the follow statements is correct when the driving angular frequency, ω_d, is varied from below the resonance frequency, ω_{res}, to above? Note the classification is based on the phase shift of the current relative to the driving EMF. Phasors may be helpful to visualize the different situations. [10 points.]

Resistor: $Z=R$ Inductor: $Z=j\omega_d L$ Capacitor: $Z=-j\frac{1}{\omega_d C}$





∴ An impedance $Z = Re(Z) + j\,Im(Z)$ is:
purely resistive if $Im(Z)=0$
more inductive if $Im(Z)>0$
more capacitive if $Im(Z)<0$

$$Z = R - j\left(\omega_d L - \frac{1}{\omega_d C}\right) = R + j\frac{L}{\omega_d}\left(\omega_d^2 - \underbrace{\frac{1}{LC}}_{\omega_{res}^2}\right) = R + j\frac{L}{\omega_d}\left(\omega_d^2 - \omega_{res}^2\right)$$

The circuit is:

(1) more inductive for $\omega_d<\omega_{res}$, more capacitive for $\omega_d>\omega_{res}$, and is inductive at $\omega_d=\omega_{res}$.

(2) more inductive for $\omega_d<\omega_{res}$, more capacitive for $\omega_d>\omega_{res}$, and is purely resistive at $\omega_d=\omega_{res}$.

(3) more inductive for $\omega_d<\omega_{res}$, more capacitive for $\omega_d>\omega_{res}$, and is capacitive at $\omega_d=\omega_{res}$.

(4) more capacitive for $\omega_d<\omega_{res}$, more inductive for $\omega_d>\omega_{res}$, and is inductive at $\omega_d=\omega_{res}$.

(5) more capacitive for $\omega_d<\omega_{res}$, more inductive for $\omega_d>\omega_{res}$, and is purely resistive at $\omega_d=\omega_{res}$.

(6) more capacitive for $\omega_d<\omega_{res}$, more inductive for $\omega_d>\omega_{res}$, and is capacitive at $\omega_d=\omega_{res}$.

(7) more inductive for $\omega_d<\omega_{res}$, purely resistive for $\omega_d>\omega_{res}$.

(8) more capacitive for $\omega_d<\omega_{res}$, purely resistive for $\omega_d>\omega_{res}$.

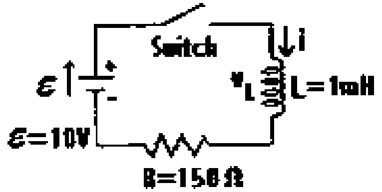
(9) purely resistive for $\omega_d<\omega_{res}$, more inductive for $\omega_d>\omega_{res}$.

(10) purely resistive for $\omega_d<\omega_{res}$, more capacitive for $\omega_d>\omega_{res}$.

$$\therefore\ Im(Z) = \frac{L}{\omega_d}\left(\omega_d^2 - \omega_{res}^2\right)\begin{cases} >0 \text{ (more inductive)} & \text{if } \omega_d > \omega_{res} \\ <0 \text{ (more capacitive)} & \text{if } \omega_d < \omega_{res} \\ =0 \text{ (purely resistive)} & \text{if } \omega_d = \omega_{res} \end{cases}$$

$\Rightarrow \boxed{(5)}$

13) At t=10μs after the switch is closed in the RL circuit below, how much energy is stored in the inductor? Here L=1mH, R=150Ω, and ε=10V. [10 points.]



(1) 0.12 μJ

(2) 0.27 μJ

(3) 1.11 μJ

(4) 1.34 μJ

(5) 17.2 μJ

(6) 3.72 μJ

(7) 1.72 μJ

(8) 5.71 μJ

(9) 12.1 μJ

(10) 0.64 μJ

After the switch has been closed a long time, final current is $i_f = \frac{\varepsilon}{R} = \frac{10\ V}{150\ \Omega} = 6.666\times10^{-2}$

Rise of current ⇒ $i(t) = i_f\left(1-e^{-t/\tau_L}\right)$

where τ_L = inductive time constant = $\frac{L}{R} = \frac{1\times10^{-3}\ H}{150\ \Omega} = 6.666\times10^{-6}\ s$
$= 6.666\ \mu s$

Energy stored in inductor at time t is

$U(t) = \frac{1}{2}L\,i(t)^2 = \frac{1}{2}L\left(i_f\left(1-e^{-t/\tau_L}\right)\right)^2$

$U(10\ \mu s) = \frac{1}{2}(1\times10^{-3}\ H)\left(6.666\times10^{-2}\left(1-e^{-\frac{10\ \mu s}{6.666\ \mu s}}\right)\right)^2$
$= 1.34\times10^{-6}\ J = 1.34\ \mu J$

$\Rightarrow \boxed{(4)}$