1. Two identical conducting spheres A and B carry equal charge. They are separated by a distance much larger than their diameters. A third identical conducting sphere C is uncharged. Sphere C is first touched to A, then to B, and finally removed. As a result, the electrostatic force between A and B, which was originally *F*, becomes:
   A) *F*/2
   B) *F*/4
   C) 3*F*/8
   D) *F*/16
   E) 0

Touching two conductors together puts them at the same potential; if the conductors are identical spheres, the total charge that was on the two spheres before they touched will be divided equally between them after they have touched.

Thus, touching C to A puts a charge of $\frac{1}{2}(\underbrace{Q}_{\text{charge on } B}+\underbrace{0}_{\text{charge on } C})=\frac{Q}{2}$ on each of A and C, where Q = initial charge on A = initial charge on B.

Touching C to B puts a charge of $\frac{1}{2}(\underbrace{\frac{Q}{2}}_{\text{charge on } C}+\underbrace{Q}_{\text{charge on } B})=\frac{3Q}{4}$ on each of B and C

Initially, force between A and B is $F=\frac{kQ^2}{r^2}$

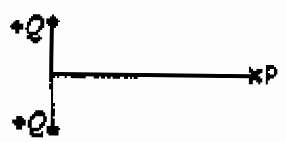
Resulting force between A and B is $F'=\frac{k\left(\frac{Q}{2}\right)\left(\frac{3Q}{4}\right)}{r^2}=\frac{3}{8}\underbrace{\frac{kQ^2}{r^2}}_{F}=\frac{3}{8}F \Rightarrow \boxed{C}$

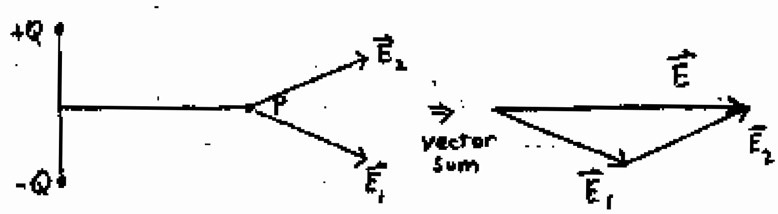
2. Charge is distributed uniformly on the surface of a spherical balloon (an insulator) with a point charge *q* inside. The electrical force on *q* is greatest when:
   A) it is near the inside surface of the balloon
   B) it is at the center of the balloon
   C) it is halfway between the balloon center and the inside surface
   D) it is anywhere inside (the force is same everywhere and is not zero)
   E) it is anywhere inside (the force is zero everywhere)

Let R be the radius of the spherical balloon. For a spherical Gaussian surface of radius $r < R$, concentric with the balloon, the charge enclosed is zero (ie $Q_{encl}(r)=0$). It follows by Gauss' Law that E(r), the magnitude of the E-field at radius r, is zero. Since this is true for any $r<R$, the $\vec{E}$-field is zero inside the balloon. It follows that the force on a charge q placed anywhere inside the balloon is $\vec{F}=q\vec{E}=0$
$\Rightarrow \boxed{E}$

3. The diagram shows two identical positive charges *Q*. The electric field at point P on the perpendicular bisector of the line joining them:

   A) ↑
   B) ↓
   C) →
   D) ←
   E) zero

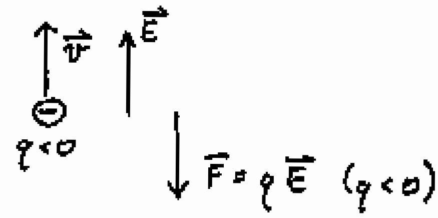




Net $\vec{E}$-field at P points to right $\Rightarrow \boxed{C}$

4. An electron traveling north enters a region where the electric field is uniform and points north. The electron:

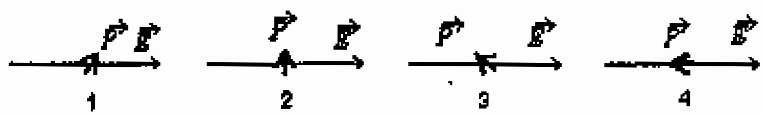
A) speeds up
B) slows down
C) veers east
D) veers west
E) continues with the same speed in the same direction



Since force $\vec{F}$ points down, the acceleration vector $\vec{a}$ also points down. The velocity vector $\vec{V}$ points up, while $\vec{a}$ points down, so the electron slows down. $\boxed{B}$

5. An electric dipole is oriented parallel to a uniform electric field, as shown.

It is rotated to one of the five orientations shown below. Rank the final orientations according to the change in the potential energy of the dipole-field system, most negative to most positive.



A) 1, 2, 3, 4
B) 4, 3, 2, 1
C) 1, 2, 4, 3
D) 3, 2 and 4 tie, then 1
E) 1, 2 and 4 tie, then 3

Electric potential energy $U_f = -\vec{p}\cdot\vec{E} = -|\vec{p}||\vec{E}|\cos\theta_f$ where θ_f is the final angle between $\vec{p}$ and $\vec{E}$.
Initial angle between $\vec{p}$ and $\vec{E}$ is $\theta_i = 0$

$$\Delta U = U_f - U_i = (-|\vec{p}||\vec{E}|\cos\theta_f) - (-|\vec{p}||\vec{E}|\underbrace{\cos\theta_i}_{1})$$

$$= |\vec{p}||\vec{E}|(1-\cos\theta_f)$$ Thus the larger the angle θ_f is the more positive ΔU is.

$\Rightarrow$ Order of increasing ΔU is 1,2,3,4 $\Rightarrow$ $\boxed{A}$

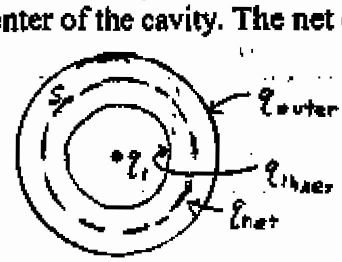
6. A point charge is placed at the center of a spherical Gaussian surface. The electric flux Φ_E is changed if:

A) the sphere is replaced by a cube of the same volume
B) the sphere is replaced by a cube of one-tenth the volume
C) the point charge is moved off center (but still inside the original sphere)
D) the point charge is moved to just outside the sphere
E) a second point charge is placed just outside the sphere

By Gauss' Law, the electric flux $\Phi_E = \frac{Q_{encl}}{\epsilon_0}$, where Q_{encl} is the charge enclosed. Only $\boxed{D}$ changes the charge enclosed, so only $\boxed{D}$ changes the electric flux.

7. 10 C of charge are placed on a spherical conducting shell. A –3-C point charge is placed at the center of the cavity. The net charge in coulombs on the inner surface of the shell is:

A) –7
B) –3
C) 0
D) +3
E) +7

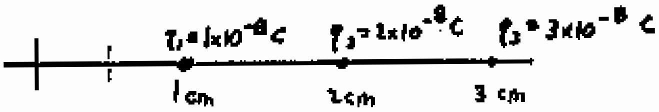


By Gauss' Law $0 = \oint_S \vec{E}\cdot d\vec{A} = \frac{\text{charge in } S}{\epsilon_0} = \frac{q_1 + q_{inner}}{\epsilon_0}$

$\vec{E}=0$ in conductor $\Rightarrow$ 0

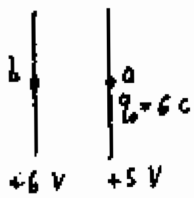
$\Rightarrow q_{inner} = -q_1 = -(-3C) = +3C \Rightarrow \boxed{D}$

8. Three charges lie on the x axis: 1×10^{-8} C at $x = 1$ cm, 2×10^{-8} C at $x = 2$ cm, and 3×10^{-8} C at $x = 3$ cm. The potential energy of this arrangement, relative to the potential energy for infinite separation, is:
A) 7.9×10^{-2} J
B) 8.5×10^{-4} J
C) 1.7×10^{-3} J
D) 0.16 J
E) zero



$$U = \sum_i \sum_{j>i} \frac{k q_i q_j}{r_{ij}} = \frac{k q_1 q_2}{r_{12}} + \frac{k q_1 q_3}{r_{13}} + \frac{k q_2 q_3}{r_{23}}$$
$$= \left(8.99\times10^{9} \frac{Nm^2}{C^2}\right)\left(\frac{(1\times10^{-8}C)(2\times10^{-8}C)}{(1\times10^{-2}m)} + \frac{(1\times10^{-8}C)(3\times10^{-8}C)}{(2\times10^{-2}m)} + \frac{(2\times10^{-8}C)(3\times10^{-8}C)}{(1\times10^{-2}m)}\right)$$
$$= 8.5\times10^{-4}\ J \Rightarrow \boxed{B}$$

9. The work in joules required to carry a 6.0-C charge from a 5.0-V equipotential surface to a 6.0-V equipotential surface and back again to the 5.0-V surface is:
A) zero
B) 1.2×10^{-5}
C) 3.0×10^{-5}
D) 6.0×10^{-5}
E) 6.0×10^{-6}



Let a be a point on 5 V surface and b be a point on 6 V surface,

$$\text{Total Work} = \underbrace{W^{ext}_{a\to b}}_{q_0(V(b)-V(a))} + \underbrace{W^{ext}_{b\to a}}_{q_0(V(a)-V(b))} = 0 \Rightarrow \boxed{A}$$

10. A hollow metal sphere is charged to a potential V. The potential at its center is:
A) V
B) 0
C) $-V$
D) $2V$
E) πV

Let R be the radius of the sphere, and let S be a Gaussian surface of radius $r < R$. Since the charge enclosed by this spherical Gaussian surface is zero, it follows by Gauss' Law that $E(r)$, the magnitude of the $\vec{E}$-field at radius r, is zero, for all $r < R$. Thus

$$\underbrace{V(r=R)}_{V} - V(r=0) = -\int_0^R \underbrace{\vec{E}}_{0}\cdot d\vec{r} = 0 \Rightarrow V(r=0) = V \Rightarrow \boxed{A}$$

11. A dielectric slab is slowly inserted between the plates of a parallel plate capacitor, while the potential difference between the plates is held constant by a battery. As it is being inserted:
A) the capacitance, the potential difference between the plates, and the charge on the positive plate all increase
B) the capacitance, the potential difference between the plates, the charge on the positive plate all decrease
C) the potential difference between the plates increases, the charge on the positive plate decreases, and the capacitance remains the same
D) the capacitance and the charge on the positive plate decrease but the potential difference between the plates remains the same
E) the capacitance and the charge on the plate increase but the potential difference between the plates remains the same

$C_{new} = \kappa C_{old}$; Since κ is always ≥ 1, this implies that $C\uparrow$.
The potential difference between the plates, V, doesn't change since it's being held constant by the battery. Since $Q = CV$, where $C\uparrow$ and V doesn't change, it follows that $Q\uparrow$. $\boxed{E}$

12. A 2-μF and a 1-μF capacitor are connected in series and charged by a battery. They store energies P and Q, respectively. When disconnected and charged separately using the same battery, they have energies R and S, respectively. Then:

A) $R > P > S > Q$
B) $P > Q > R > S$
C) $R > P > Q > S$
D) $P > R > S > Q$
E) $R > S > Q > P$

Capacitors in series have the same charge on each. If Q is the charge on each capacitor when in series, then since the energy stored in a capacitor is $U = Q^2/2C$, the smaller capacitor will store more energy, so $\boxed{Q > P}$. The voltage drop across either capacitor when the two capacitors are in series is smaller than when the battery is connected directly across that capacitor (since for the series combination the sum of the voltage drops across each capacitor equals the voltage of the battery), so since the energy stored in a capacitor is $U = \frac{1}{2}CV^2$, it follows that $\boxed{(R, S) > (Q, P)}$. When the two capacitors are connected directly across the battery, the larger capacitor stores more energy (since $U = \frac{1}{2}CV^2$), so $\boxed{R > S}$. From the foregoing, $\boxed{R > S > Q > P}$. $\therefore$ $\boxed{E}$

13. An ordinary light bulb is marked "60 watt, 120 volt". Its (heated) resistance is:

A) 60 Ω
B) 120 Ω
C) 180 Ω
D) 240 Ω
E) 15 Ω

$P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P} = \frac{(120\ V)^2}{60\ W} = 240\ \Omega$. $\therefore$ $\boxed{D}$

14. Nine identical wires, each of diameter d and length L, are connected in series. The combination has the same resistance as a single similar wire of length L but whose diameter is:

A) $3d$
B) $9d$
C) $d/3$
D) $d/9$
E) $d/81$

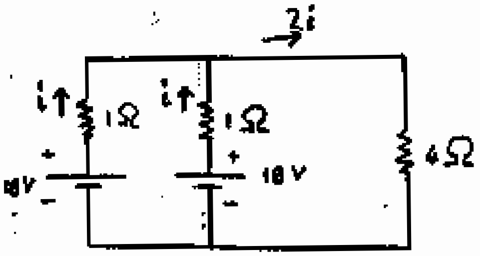
Resistors in series add $\Rightarrow R = 9\frac{\rho L}{A} = 9\frac{\rho L}{\pi (d/2)^2} = \frac{1}{(\frac{1}{3})^2}\frac{\rho L}{\pi (d/2)^2}$

$= \frac{\rho L}{\pi\left(\left(\frac{d/3}{2}\right)\right)^2}$

$\Rightarrow$ same resistance as one wire of length L and diameter $\frac{d}{3}$. $\therefore$ $\boxed{C}$

15. Two identical batteries, each with an emf of 18 V and an internal resistance of 1 Ω, are wired in parallel by connecting their positive terminals together and connecting their negative terminals together. The combination is then wired across a 4-Ω resistor. The current in each battery is:

A) 1.0 A
B) 2.0 A
C) 4.0 A
D) 3.6 A
E) 7.2 A



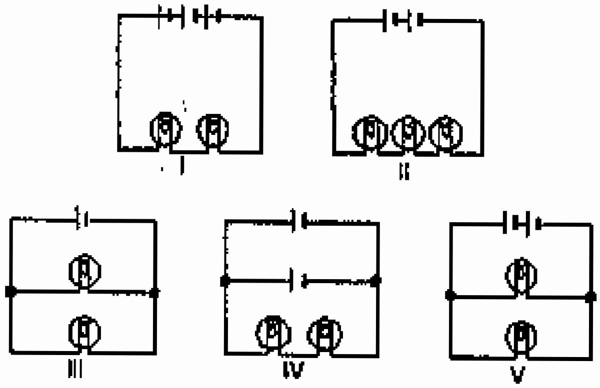
By symmetry, the current thru each battery is the same.
Let i be the current thru one of the batteries.

loop equation for right loop:

$18 - i(1\Omega) - 2i(4\Omega) = 0$

$\Rightarrow 18 = 9i \Rightarrow i = 2A \Rightarrow \boxed{B}$

16. In the diagrams, all light bulbs are identical and all emf devices are identical. In which circuit (I, II, III, IV, V) will the bulbs be dimmest?



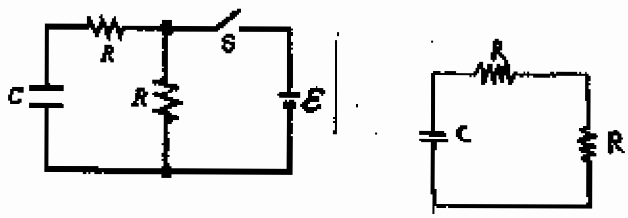
A) I
B) II
C) III
D) IV
E) V

Since a light bulb can be thought of as a resistor, and for a resistor the power dissipated is $P = i^2R$, we are looking for the circuit with the least current thru the bulbs. Let V be the voltage of one battery, R the resistance of one lamp, and i the current thru one lamp. Writing a loop equation for each circuit:

I: $3V - iR - iR = 0 \Rightarrow i = 3V/2R$
II: $2V - iR - iR - iR = 0 \Rightarrow i = 2V/3R$
III: $V - iR = 0 \Rightarrow i = V/R$
IV: $V - iR - iR = 0 \Rightarrow i = V/2R$ ⟵ smallest i ⇒ dimmest ⇒ $\boxed{D}$
V: $2V - iR = 0 \Rightarrow i = 2V/R$

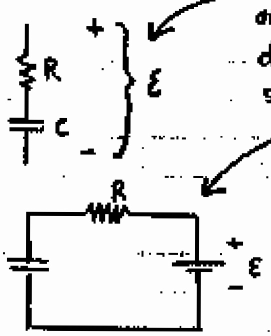
17. In the circuit shown, both resistors have the same value R. Suppose switch S is initially closed. When it is then opened, the circuit has a time constant τ_a Conversely, suppose S is initially open. When it is then closed, the circuit has a time constant τ_b The ratio τ_a/τ_b is:

A) 1
B) 2
C) 0.5
D) 0.667
E) 1.5



When the switch is initially closed, the battery loads charge onto the plates of the capacitor. Then when the switch is opened, the capacitor discharges thru the two resistors, as shown above. The time constant of the circuit is $\tau_a = CR_{eq} = C(R+R) = 2RC$.

If the switch is initially open and is then closed, the battery begins charging the capacitor. The battery forces the voltage across the branch of the circuit containing the capacitor to be $\mathcal{E}$, as shown below; this determines the rate at which the capacitor charges, and hence also determines the time constant. Thus for the purpose of determining the time constant, we may as well have the circuit shown below; the time constant for this circuit is $\tau_b = RC$



$\therefore \frac{\tau_a}{\tau_b} = \frac{2RC}{RC} = 2 \Rightarrow \boxed{B}$