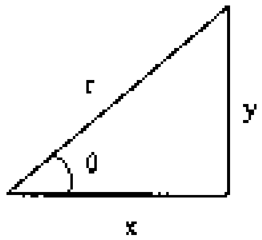
Key

# Physics 220 – Exam #3

November 15 2001

This exam consists of 12 problems on 7 pages. Please check that you have them all.

All of the formulas that you will need are given below. You may also use a calculator.




$\sin\theta = y/r$ $\cos\theta = x/r$ $\tan\theta = y/x$

average speed $= \frac{\text{distance traveled}}{\text{time}}$ $g = 9.8\ \text{m/s}^2$

average velocity $= \bar{v} = \frac{\text{displacement}}{\text{time}}$

instantaneous velocity = slope of position versus time
instantaneous acceleration = slope of velocity versus time

For constant acceleration:

$x = x_0 + v_0t + \frac{1}{2}at^2$ $v = v_0 + at$ $v^2 = v_0^2 + 2a(x - x_0)$

$\vec{F} = m\vec{a}$ $F^{max}_{friction} = \mu_S N$ (static friction) $F_{friction} = \mu_K N$ (sliding friction)

$F_{gravity} = \frac{Gm_1m_2}{r^2}$ $G = 6.67 \times 10^{-11}\ \text{Nm}^2/\text{kg}^2$

$a_c = v^2/r$ $KE = \frac{1}{2}mv^2$ $W = F\,d\,\cos\theta$ $PE_{gravity} = mgh$

power $=$ work$/\Delta t$ $\vec{p} = m\vec{v}$ $\Delta p =$ impulse $= F\,\Delta t$

$F_{spring} = -kx$ $PE_{spring} = \frac{1}{2}kx^2$ $x = A\sin(\omega t)$ $v = A\omega\cos(\omega t)$

$\omega = \sqrt{k/m}$ $\omega = \sqrt{g/L}$ $\omega = 2\pi f$ $f = 1/T$

For constant angular acceleration:

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$ $\omega = \omega_0 + \alpha t$ $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

$\text{KE} = I\omega^2/2$ $L = I\omega$ $\tau = I\alpha$ $\theta = s/r$ $\omega = v/r$ $\alpha = a/r$ $\omega = \theta/t$

Pressure $=$ Force/area $A_1v_1 = A_2v_2$ $P_1 + \rho gh_1 + \rho v_1^2/2 = P_2 + \rho gh_2 + \rho v_2^2/2$

Archimedes principle: buoyant force = weight of fluid displaced

1. A record of diameter 0.12 m is initially at rest. The record player is then turned on and the record accelerates uniformly to an angular speed of 3.5 radians/s. If the record makes 5.5 revolutions during this time, find the angular acceleration of the record.

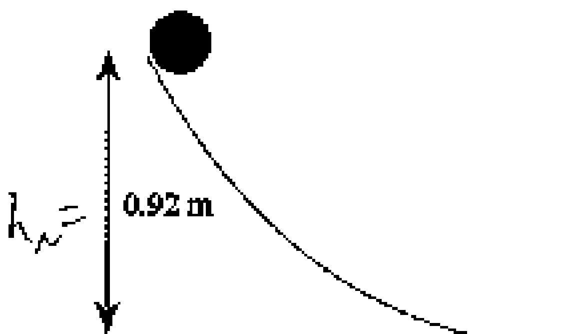
(a) 1.1 rad./s$^2$
(b) 0.64 rad./s$^2$
(c) 0.18 rad./s$^2$
(d) 0.10 rad./s$^2$
(e) 2.8 rad./s$^2$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\omega_0 = 0$$

$$\alpha = \frac{\omega^2}{2\theta} = \frac{(3.5)^2}{2(5.5\cdot 2\pi)} = 0.18\,\text{rad/s}^2$$

2. A sphere of mass 3.5 kg rolls without slipping down a ramp of height 0.92 m as shown below. Find the speed of the sphere when it reaches the bottom of the ramp. Ignore air drag. The moment of inertial of a sphere is $2mR^2/5$.




(a) 18 m/s
(b) 4.2 m/s
(c) 32 m/s
(d) 0.29 m/s
(e) 3.6 m/s

$$KE_i + PE_i = KE_f + PE_f$$

$$\tfrac{1}{2} m v_i^2 + \tfrac{1}{2} I \omega_i^2 + m g h_i = \tfrac{1}{2} m v_f^2 + \tfrac{1}{2} I \omega_f^2 + m g h_f$$

$$v_i = 0, \quad \omega_i = 0, \quad h_f = 0$$

$$m g h_i = \tfrac{1}{2} m v_f^2 + \tfrac{1}{2} I \omega_f^2 = \tfrac{1}{2} m v_f^2 + \tfrac{1}{2}\cdot\tfrac{2}{5} m R^2 \frac{v_f^2}{R^2}$$

$$g h_i = \tfrac{7}{10} v_f^2$$

$$v_f = \sqrt{\tfrac{10}{7} g h_i} = \sqrt{\tfrac{10}{7}(9.8)(0.92)} = 3.6\,\text{m/s}$$

3. A particle attached to a spring with spring constant $k = 125\,\mathrm{N/m}$ is undergoing simple harmonic motion, and its position is given by the equation $x = 1.7\cos(24t)$. Find the period of the motion.

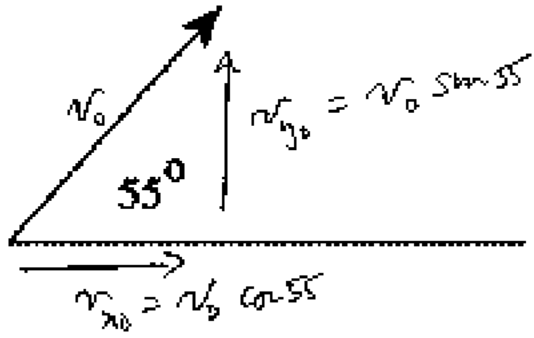
   (a) 24 s
   (b) 0.042 s
   (c) 0.083 s
   (d) 1.6 s
   (e) 0.26 s

$X = A\cos(\omega t)$

24

$\omega = \frac{2\pi}{T}$ $\quad T = \frac{2\pi}{\omega} = \frac{2\pi}{24} = 0.26\,s$

4. A baseball is hit with an initial speed of 35 m/s at an angle of 55° with respect to the (horizontal) ground. How far from home plate does the ball land? Ignore air drag, and assume that the ball starts at ground level.




   (a) 5.9 m
   (b) 120 m
   (c) 340 m
   (d) 59 m
   (e) 85 m

$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$

lands when $\Rightarrow$ $0 = 0 + v_{y0}t - \frac{1}{2}gt^2$ $\quad t = \frac{2 v_{y0}}{g}$

$X = x_0 + v_{x0}t = v_{x0} \cdot \frac{2 v_{y0}}{g} = \frac{2 v_0^2 \sin 55 \cos 55}{g}$

$= 118\,m$

5. A block of mass 4.4 kg is dropped from a height of 3.3 m onto a spring that is initially at rest (i.e., unstretched and uncompressed). When the block lands on the spring, the spring is compressed an amount 0.62 m before coming momentarily to rest. Find the spring constant $k$ of the spring.

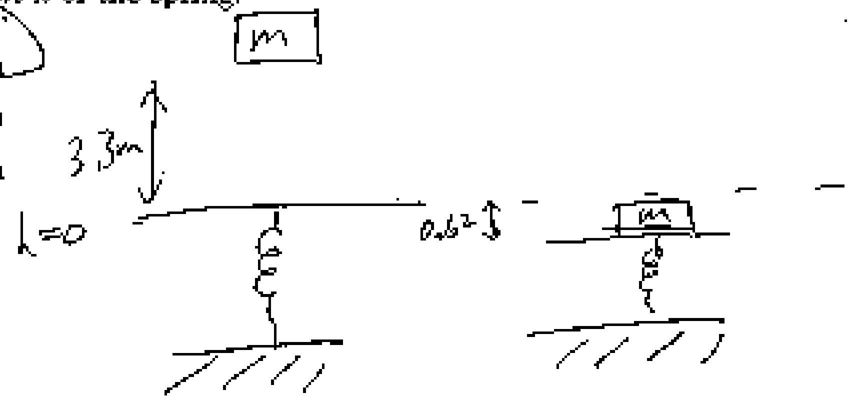
(a) 880 N/m

(b) 250 N/m

(c) 540 N/m

(d) 11 N/m

(e) 70 N/m




$KE_i + PE_i = KE_f + PE_f$

$\frac{1}{2}mv_i^2 + mgh_i + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + mgh_f + \frac{1}{2}kx_f^2$

$mgh_i = mgh_f + \frac{1}{2}kx_f^2$

$\frac{1}{2}kx_f^2 = mg(h_i - h_f)$

$k = \frac{2mg(h_i - h_f)}{x_f^2} = \frac{2(4.4)9.8(3.3+0.62)}{0.62^2}$

$= \underline{880\ N/m}$

6. Your best friend has a powerful bow (for archery). The bow-string combination acts as a spring with spring constant $k = 400$ N/m. You have an arrow of mass 0.15 kg. If you want to shoot the arrow with a speed of 65 m/s, through what distance must you pull back the bow string?

(a) 0.049 m

(b) 0.69 m

(c) 1.3 m

(d) 0.89 m

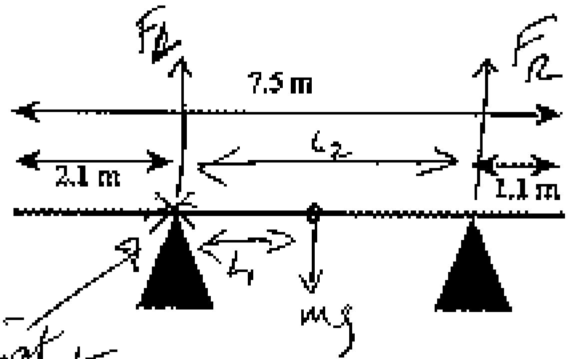
(e) 0.25 m

$\frac{1}{2}mv_i^2 + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}kx_f^2$

$\frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2$

$x_i = v_f\sqrt{\frac{m}{k}} = 65\sqrt{\frac{0.15}{400}} = \underline{1.3m}$

7. A plank of length 7.5 m and mass 55 kg sits at rest on two supports as shown below. The mass of the is distributed uniformly. Find the force of the right support on the plank.




(a) 210 N
(b) 540 N
(c) 79 N
(d) 470 N
(e) 270 N

$\sum \tau = 0 = mgL_1 - F_R L_2 \qquad F_R = mg\frac{L_1}{L_2}$

$L_2 = 7.5 - 2.1 - 1.1 = 4.3$

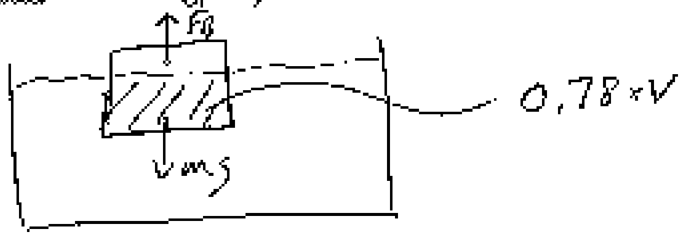
$L_1 = 3.75 - 2.1 = 1.65$

$F_R = 55(9.8)\frac{1.65}{4.3}$

$= \underline{210\ N}$

8. A box of volume 4.3 m$^3$ is floating in a lake. 78% of the box is under water. Find the mass of the box. ($\rho_{water} = 1000$ kg/m$^3$)

(a) 3400 kg
(b) 780 kg
(c) 1000 kg
(d) 2700 kg
(e) 4300 kg




$\sum F = 0 = F_B - mg$

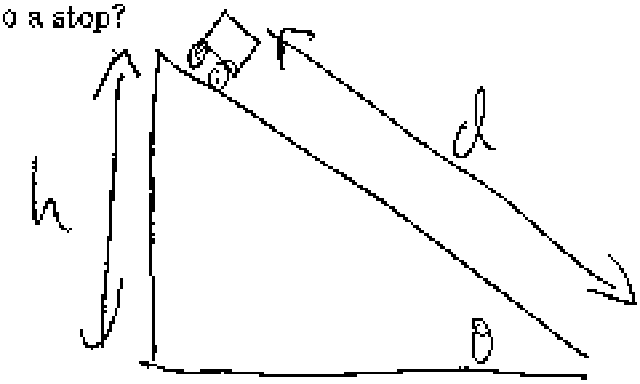
$= \rho_{water} \cdot (0.78V)g - mg$

$m\cancel{g} = \rho(0.78)V\cancel{g}$

$m = 1000(0.78)4.3$

$= \underline{3400\ kg}$

9. A bus is traveling at 12 m/s down a hill when the brakes are applied and all four wheels lock. The hill is an inclined plane that makes an angle of 17° with the horizontal. The coefficient of friction between the tires and the road is 0.60. How far does the bus skid before coming to a stop?



(a) 7.3 m

(b) 26 m

(c) 13 m

(d) 52 m

(e) 93 m

$$\frac{1}{2} m v_i^2 + mgh - W_{fric} = KE_f + PE_f = 0$$

$$\frac{1}{2} m v_i^2 = \mu N d - mgh = \mu mg\cos\theta\, d - mg\, d\sin\theta$$

$$\frac{v_i^2}{2} = d \cdot g(\mu\cos\theta - \sin\theta$$

$$d = \frac{v_i^2}{2g(\mu\cos\theta - \sin\theta)} = \underline{\underline{26 m}}$$

10. An airplane wing is designed to operate with an air speed of 300 m/s over the top of the wing and 225 m/s across the bottom. If the lift force on the wing is $5.6 \times 10^5$ N, find the area of the wing. (Helpful hint: you may ignore the thickness of the wing.) ($\rho_{air} = 1.3$ kg/m$^3$; $P_{atm} = 1.0 \times 10^5$ Pa)

(a) 9.6 m$^2$

(b) $1.1 \times 10^4$ m$^2$

(c) 22 m$^2$

(d) 41 m$^2$

(e) 87 m$^2$

$$P_1 + \cancel{\rho g h_1} + \frac{1}{2}\rho v_1^2 = P_2 + \cancel{\rho g h_2} + \frac{1}{2}\rho v_2^2$$

$$h_1 \approx h_2$$

$$F = (P_1 - P_2)A = \left(\frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2\right)A$$

$$= \frac{1}{2}\rho(v_2^2 - v_1^2)A$$

$$A = \frac{2F}{\rho}\frac{1}{v_2^2 - v_1^2} = \frac{2 \cdot 5.6\times10^5}{1.3}\frac{1}{300^2 - 225^2}$$

$$= \underline{\underline{22 m^2}}$$

11. The water pressure in the pipes in your house is $3.5 \times 10^5$ Pa. When you open a faucet, what is the speed of the water when it sprays out? Assume that the opening of the faucet is very small. ($\rho_{water} = 1000$ kg/m$^3$; $\rho_{air} = 1.3$ kg/m$^3$; $P_{atm} = 1.0 \times 10^5$ Pa)

(a) 500 m/s
(b) 22 m/s
(c) 16 m/s
(d) 26 m/s
(e) 6.9 m/s

$$P_1 + \frac{1}{2}\rho v_1^2 + \cancel{\rho g h_1} = P_2 + \frac{1}{2}\rho v_2^2 + \cancel{\rho g h_2}$$

$P_1 = 3.5\times10^5$, $v_1 = 0$, $P_2 = 1.0\times10^5$

$$1.0\times10^5 + \frac{1}{2}\rho v_2^2$$

$$\frac{1}{2}\rho v_2^2 = (3.5-1.0)\times10^5 = 2.5\times10^5$$

$$v_2 = \sqrt{\frac{2(2.5\times10^5)}{1000}} = \underline{\underline{22}}$$

12. Your job is to design a pendulum clock using a simple pendulum (i.e., a mass tied to a string). The angular frequency of a simple pendulum is $\omega = \sqrt{g/L}$ where $L$ is the length of the pendulum. You want your clock to make one "tick" (one period) every 5.0 s. Find the value of $L$ that is required.

(a) 0.65 m
(b) 8.9 m
(c) 1.2 m
(d) 0.25 m
(e) 6.2 m

$$\omega = \sqrt{\frac{g}{L}} = \frac{2\pi}{T}$$

$$\frac{g}{L} = \frac{4\pi^2}{T^2}$$

$$L = \frac{gT^2}{4\pi^2} = \frac{9.8(5)^2}{4\pi^2} = \underline{\underline{6.2\,m}}$$

The End