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PHYS 172 - Fall 2011

Name (Print):______________________________

Signature:______________________________

PUID:______________________________

Write down your recitation time:

Day (either Wed, Th, Fri):

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Time:

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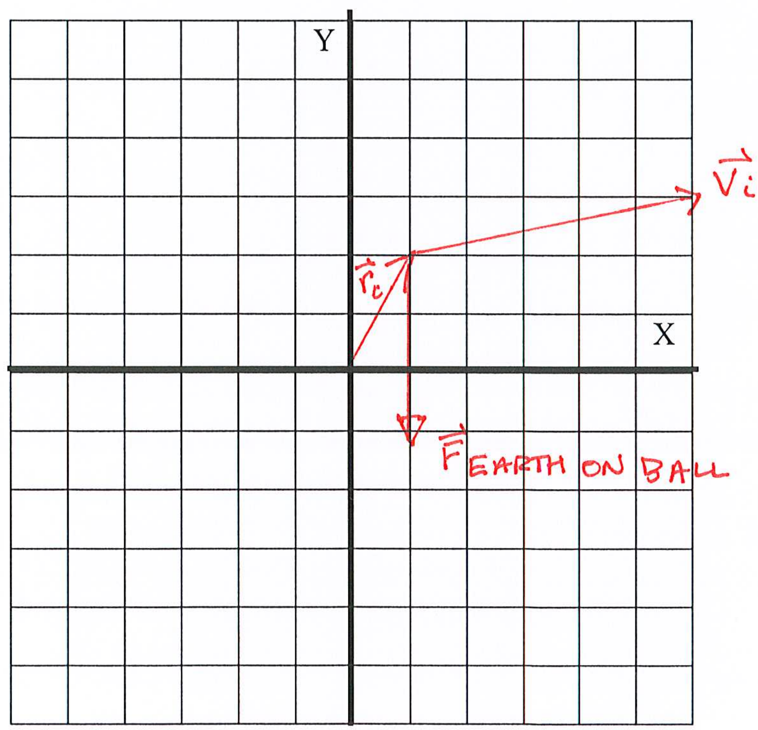
Show as much work as possible to get full credit: list what you know, draw diagrams, define the system, list the relevant physical principle, and then solve the equation.

You throw a 0.2 kg ball from shoulder height into the air. Right after it leaves your hand, the ball's position is $\vec{r}_i = \langle 1,2,0 \rangle\, m$ and its initial velocity is $\vec{v}_i = \langle 10,2,0 \rangle\, m/s$. We want to determine the position of the ball after 0.3 seconds. Your feet are located at <0,0,0>. Take the ball to be the "system" for parts 1-3, and treat air resistance as negligible.

1. [3 points] Draw the initial state of the system, using a vector to indicate the ball's initial position, a vector to indicate the ball's initial velocity, and a vector to indicate the direction of the gravitational force due to the Earth on the ball.



2. [8 points] What is the momentum of the ball after 0.3 seconds? What principle will you use in order to answer this question?

Momentum Principle: $\Delta\vec{p} = \vec{F}_{net}\Delta t$ ← 2 pts

$\vec{p}_f = \vec{p}_i + \vec{F}_{net}\Delta t = m \cdot \vec{v}_i + \vec{F}_{net}\Delta t$, **and** $|\vec{v}_i| \ll c$ ← 2 pts

$\vec{p}_f = m\langle v_{i,x}, v_{i,y}, 0\rangle + \langle 0, -mg, 0\rangle\Delta t = \langle 2, -0.2, 0\rangle\ kg \cdot m/s$

3. [8 points] What is the position of the ball after 0.3 seconds? What principle will you use in order to answer this question?

Position update principle $\Delta\vec{r} = \vec{v}_{ave}\Delta t$, $\vec{r}_r = \vec{r}_i + \dfrac{\vec{v}_i + \vec{v}_f}{2}\Delta t$

This expression for $\vec{v}_{ave}$ is appropriate for a constant force or a time interval over which the force can be treated as constant.

$$\vec{r}_f = \langle x_i, y_i, 0\rangle + \frac{1}{2}\left\langle v_{i,x} + \frac{p_{f,x}}{m}, v_{i,y} + \frac{p_{f,y}}{m}, 0\right\rangle\Delta t$$

$$\vec{r}_f = \langle 1,2,0\rangle + \frac{1}{2}\left\langle 10 + \frac{2}{0.2}, 2 + \frac{-0.2}{.2}, 0\right\rangle \cdot 0.3\ m$$

$$\vec{r}_f = \langle 4, 2.15, 0\rangle\ m$$

4. [11 points] Now assume that you are wearing ice-skates and are standing on frictionless ice. What is your velocity right after releasing the ball? Give the full vector form. Take your mass, *M*, to be 50 kg. Identify the object(s) that are in the system and in the surroundings and the forces that act on the object(s). What principle will you use to answer this question?

System 1: You, ball & Earth, ice.
Forces: All forces are internal to the system. Reciprocity leads us to describe the forces in terms of pairs:

Force you exert on ball – force ball exerts on you, components in x and y-direction.
Force Earth exerts on ball - force ball exerts on Earth – y-component only.
Force Earth exerts on you – force you exert on Earth – y-component only.
Force ice exerts on you – force you exert on ice – y-component only.
There is nothing in the surroundings acting on the system.

Principle: Conservation of Momentum. The momentum of the system is constant (zero). You do not crash through the ice upon releasing the ball, so the net force on you in the y-direction is zero. (Because the mass of the Earth is so much greater than that of the ball, we need only concern ourselves with the forces acting in the x-direction.)

$p_{ball,x} = -p_{you,x} \quad p_{ball,x} = 0.2 \cdot 2\ kg \cdot m/s = -50 \cdot v_{you,x}$

$v_{you,x} = -.04\ m/s$

$\vec{v}_{you} = \langle -0.04, 0, 0 \rangle\ m/s$

System 2: You and Ball
Surroundings: Earth, ice

Forces acting between objects within the system: Force you exert on ball upon throwing it, force of ball on you (x and y-components).

Forces due to interactions of surroundings with objects in the system: Gravitational force of Earth on You and on Ball - y-direction.
Force of ice on you (y-direction)

You don't acquire momentum in the y-direction, so the net force acting on you in the y-direction is zero. No interaction of surroundings has a component of force acting on you in the x-direction.

The Momentum Principle requires that momentum in the x-direction be conserved because the sum of the forces in the x-direction is zero (reciprocity). When the ball leaves your hand, it has an initial momentum. You must recoil with x-momentum of an equivalent magnitude.

$$\vec{p}_{you} = \langle -p_{ball,x,i}, 0, 0 \rangle$$
$$\vec{v}_{you} = \left\langle \frac{-2}{50}, 0, 0 \right\rangle m/s = \langle -0.04, 0, 0 \rangle\ m/s$$

System 3: You
Surroundings: Earth, Ice, Ball
Force acting on you:
x-direction: Force of ball on you due to change of ball's x-component of momentum.
y-direction: Normal force (feet on ice), gravity, force of ball on you (in y-direction)

You don't accelerate in the y-direction, so the net force on you in the y-direction must be zero. Only the x-component of the net force on you need be considered. This must be equal and opposite the force in the x-direction that you exerted on the ball.

$$\vec{p}_{you} = \langle -F_{ball,x}, 0, 0 \rangle \Delta t = \langle -p_{ball,x}, 0, 0 \rangle$$

$$\vec{v}_f = \left\langle \frac{-2}{50}, 0, 0 \right\rangle m/s = \langle -0.04, 0, 0 \rangle\ m/s$$