1. (6 points) In the circuit shown below the switch has been open for a long time. It is then suddenly closed. What is the current through the switch immediately when it is closed?

\[ V = 20 \text{ V}, \quad R_1 = 50 \text{ k}\Omega, \quad R_2 = 100 \text{ k}\Omega \]

\[ i = \frac{V}{R_1} + \frac{V}{R_2} = \frac{20}{50} + \frac{20}{100} = 0.4 + 0.2 = 0.6 \text{ A} \]

2. (6 points) A 40 μC charge is located on the z axis at \( z = 4 \text{ cm} \). At what \( z \) should a \(-60 \mu C\) charge be placed to produce a net electric field of zero at the origin \( (z = 0) \)?

\[ E_z = \frac{q_1}{4\pi\varepsilon_0 r_1^2} - \frac{q_2}{4\pi\varepsilon_0 r_2^2} = 0 \]

\[ \frac{40}{4\pi\varepsilon_0 (0.04)^2} - \frac{60}{4\pi\varepsilon_0 z^2} = 0 \]

\[ z = \frac{2.5}{3} \text{ cm} \approx 0.83 \text{ cm} \]

3. (6 points) A wire of diameter \( D \), length \( L \), and resistance \( R \) is replaced in a circuit by a wire of the same material having a diameter \( D/2 \) and length 2\( L \). The resistance of the new wire is:

\[ R' = \rho \frac{2L}{\pi (D/2)^2} = \frac{2}{\pi} \frac{L}{D^2} \rho = \frac{1}{4} \rho \]

4. (6 points) The maximum capacitance that one can obtain by connecting together (either in series, or parallel or a mixed combination) three capacitors of values 1.0 μF, 2.0 μF, and 2.0 μF is:

\[ C_{\text{max}} = \frac{1}{\frac{1}{1.0} + \frac{1}{2.0} + \frac{1}{2.0}} = \frac{1}{\frac{1}{1.0} + \frac{1}{2.0}} = \frac{1}{0.5} = 2.0 \mu F \]

5. (6 points) The magnetic flux through a coil of resistance 30 Ω is changed from 5 \times 10^{-4} Φ·m² to 5 \times 10^{-4} Φ·m² in 0.1 second. How much charge will flow through the coil during this time?

\[ \text{flux change} = 5 \times 10^{-4} Φ \cdot m² - 5 \times 10^{-4} Φ \cdot m² = 0 \]

\[ \text{flux change} = 0 \]

\[ i = \frac{\text{flux change}}{\text{resistance}} = \frac{0}{30} = 0 \]

\[ Q = i \cdot \Delta t = 0 \cdot 0.1 = 0 \]

\[ Q = 0 \]
6. (8 points) Two straight, very long, parallel conductors carry currents $I_1$ and $I_2$ in the directions as shown in the figure below. If the magnetic field at point $A$ is to be zero, the ratio of the currents $I_2/I_1$ must be:

\[
\frac{\mu_0 I_2}{2\pi r} = \frac{\mu_0 I_1}{2\pi (d + r)}
\]

\[
\Rightarrow \frac{I_2}{I_1} = \frac{d}{2r - d}
\]

7. (8 points) An electron follows a circular path (of radius $r = 15 \text{ cm}$) in a uniform magnetic field $B = 3 \times 10^{-4} \text{ T}$. What is the period of the circular motion?

\[
\text{angular velocity} \quad \omega = \frac{qB}{m_e} \Rightarrow \frac{2\pi}{T} = \frac{qB}{m_e} \Rightarrow T = \frac{2\pi m_e}{qB}
\]

\[
T = \frac{2\pi (9.11 \times 10^{-31} \text{ kg})}{(1.6 \times 10^{-19} \text{ C}) (3 \times 10^{-4} \text{ T})} = 1.19 \times 10^{-8} \text{ s}
\]

8. (8 points) A sinusoidal voltage $V(t) = (40 \text{ V}) \sin(100t)$ is applied to a series LCR circuit with $L = 200 \text{ mH}$, $C = 50 \mu\text{F}$, and $R = 120 \Omega$. The phase angle $\phi$ between the current and the voltage is:

\[
\cos \phi = \frac{L}{R} \Rightarrow \phi = \arccos \left( \frac{L}{R} \right)
\]

\[
\Rightarrow \phi = \arccos \left( \frac{200 \text{ mH}}{120 \Omega} \right)
\]

\[
\Rightarrow \phi = 56.3^\circ
\]

9. (8 points) A very long straight wire has a charge uniformly distributed over its length. The electric field at a distance of 10 cm from the wire has a magnitude of $1.63 \times 10^7 \text{ N/C}$ and is directed opposite the wire and is perpendicular to the wire. The linear charge density on the wire is:

\[
\lambda = \frac{q}{l} = \frac{1.63 \times 10^7 \text{ N/C} \times 10^{-2} \text{ m}}{2 \pi 	imes 10^{-2} \text{ m}} = 2.6 \times 10^7 \text{ C/m}
\]

10. (5 points) The potential difference $V$ between the two plates of a parallel-plate capacitor is maintained at a constant value by connecting it to a battery. If the plates are pulled apart (maintaining the constant potential difference between the plates) how does the magnitude of the charge on the plates change?

\[
\text{For parallel plate capacitor}
\]

\[
\frac{Q}{V} = \frac{C}{A} \Rightarrow \frac{dQ}{dV} \Rightarrow V \text{ changes due to area change}
\]

\[
\Rightarrow \Delta Q = \frac{C}{A} \Delta V
\]

\[
\Delta Q = \frac{C}{A} \Delta V
\]

\[
\Rightarrow \Delta Q = \frac{CV - 100}{A} \Delta V
\]

\[
\Rightarrow \Delta Q = \frac{C}{A} \Delta V
\]
11. (9 points) When the switch S in the circuit shown below is thrown closed, the current takes 3.00 s to reach 95% of its final value. If \( R = 20.0 \, \Omega \), what is the value of the inductance \( L \)?

\[
L = \frac{R}{\ln(\frac{1 - 0.95}{0.95})} = \frac{20.0}{\ln(\frac{1 - 0.95}{0.95})} = 200 \, \text{mH}
\]

12. (8 points) A wire carries a steady current of 2.5 A. A straight portion of the wire is 1.00 m long and lies along the z axis within a magnetic field \( B = (1.6k) \, \text{T} \). If the current is in the +z direction, the magnetic force on the section of the wire is:

\[
F = i \times B = (2.5 \, \text{A})(1.00 \, \text{m})(1.6 \, \text{k}) = 4 \, \text{N}
\]

- (A) \( 4 \, \text{N} \)
- (B) \( 6 \, \text{N} \)
- (C) \( 14 \, \text{N} \)
- (D) \( 12 \, \text{N} \)
- (E) \( -10 \, \text{N} \)

13. (8 points) A bright point source of light is placed at the bottom of a large swimming pool. If the depth of the water in the pool is 2.0 m, what is the radius of the largest circle at the surface of water through which light can emerge?

\[
r = \frac{D}{2} \times \sin^{-1}(\frac{D}{2}) = \frac{2.0}{2} \times \sin^{-1}(\frac{2.0}{2}) = 2.29 \, \text{m}
\]

14. (8 points) A light beam in air is incident on a piece of glass. Which of the properties of the light are changed upon entering the glass?

- (A) the frequency, velocity, and wavelength
- (B) the frequency and the velocity
- (C) the velocity and the wavelength
- (D) the velocity only
- (E) the frequency and the wavelength

15. (8 points) A light ray initially in air enters a transparent plastic material at an angle of incidence of 60°, and the transmitted ray (inside the plastic) is refracted at an angle of 34°. What is the speed of light in the plastic material?

\[
\eta = \frac{n_1 \sin \theta_1}{n_2 \sin \theta_2} = \frac{1.23 \times 10^8 \, \text{m/s}}{3.75 \times 10^8 \, \text{m/s}} = 0.33
\]

\[
\eta_2 = \frac{1}{\eta_1} \sin^{-1}(\frac{3.75 \times 10^8 \, \text{m/s}}{1.23 \times 10^8 \, \text{m/s}}) = 1.05\pi
\]

\[
\eta_1 = \frac{n_1 \sin \theta_1}{n_2 \sin \theta_2} = \frac{1.23 \times 10^8 \, \text{m/s}}{3.75 \times 10^8 \, \text{m/s}} = 0.33
\]

\[
\eta_2 = \frac{1}{\eta_1} \sin^{-1}(\frac{3.75 \times 10^8 \, \text{m/s}}{1.23 \times 10^8 \, \text{m/s}}) = 1.05\pi
\]

\[
\eta_1 = \frac{n_1 \sin \theta_1}{n_2 \sin \theta_2} = \frac{1.23 \times 10^8 \, \text{m/s}}{3.75 \times 10^8 \, \text{m/s}} = 0.33
\]
21. (8 points) The electric field in an electromagnetic wave (in SI units) is described by the equation:

\[ E = 9(1800) \sin \left( \frac{6 \times 10^7 \pi}{2} - \omega t \right) \]

The magnitude and direction of the corresponding magnetic field is:

\[ B = \frac{E}{c} = 9 \times 10^{-14} \text{ T} \text{m}^2/\text{V} \]

22. (8 points) You are standing 50 cm in front of a reflective spherical Christmas tree ornament 8 cm in diameter. The location of your pupil is:

(A) 2.08 cm, in front
(B) 2.08 cm, behind
(C) 1.83 cm, in front
(D) 1.83 cm, behind
(E) 4.00 cm, in front

23. (8 points) A laser beam with wavelength \( \lambda = 471 \text{ nm} \) is incident upon two slits 0.2 mm apart. How far will the third dark fringe be from the central maximum, if the interference pattern is observed on a screen 5 m from the slits?

(A) 41.2 mm
(B) 37.7 mm
(C) 23.6 mm
(D) 18.5 mm
(E) 12.8 mm

24. (8 points) Two spectral lines in the sodium yellow doublet have the wavelengths of 589.592 nm and 589.392 nm respectively. What is the minimum number of lines a diffraction grating must have to resolve these two lines in first order?

(A) 780
(B) 907
(C) 1326
(D) 2528
(E) 3602

To resolve to first order (m = 1), have

\[ R = N \lambda/d \]

so minimum number of lines is

\[ N = 787 \]
16. (6 points) Five identical resistors, each of value R, are connected as shown in the figure below. If the resistance between the terminals a and b is 160 Ω, what is the value of R?

\[ R_{total} = \frac{R \times R}{R + R} = \frac{2R}{3} \]

\[ R_{total} = 160 \Omega \]

\[ R = 120 \Omega \]

17. (6 points) Nibium metal becomes superconducting when cooled below 9 K. If the superconductivity is destroyed when the surface magnetic field exceeds 0.10 T, determine the maximum current a 2.4-mm-diameter wire can carry and remain superconducting.

\[ i = \frac{B_{max} \times \pi \times r^2}{\sigma} = \frac{12 \text{ mm}}{\pi} = 1.2 \text{ mm} \]

18. (6 points) Sunlight falls on a thin oil film (index of refraction \( n = 1.44 \)) which is floating on water (index of refraction \( n = 1.33 \)). The thickness of the film is 250 nm. What color does this film appear when viewed from directly above? (The numbers in the parentheses refer to the wavelength in nm.)

\[ \lambda = \frac{2d}{n_t - n_f} = \frac{2(250 \text{ nm})}{1.44 - 1.33} \]

\[ n_t = 1.44, \quad n_f = 1.33 \]

\[ \lambda = 670 \text{ nm} \]

19. (6 points) The single slit diffraction pattern produced by a slit 2.5 x 10^{-4} m in width, illuminated by the light of wavelength 655 nm is observed on a screen. If the distance between the first and third minima is the minimum pattern is 8.0 mm, how far is the screen from the slit?

\[ \Delta y_1 - \Delta y_3 = \frac{2d}{x} \]

\[ \Delta y_1 = \frac{2d}{x} \]

\[ \Delta y_3 = \frac{2d}{x} \]

\[ x = \frac{\lambda}{d} \]

20. (6 points) Light containing colors red, yellow, violet, blue, and green falls on a diffraction grating. Which color has maximum to the central spot?

\[ \text{Red} \]

For a diffraction grating, the angle \( \theta \) for the first-order maximum matches with

\[ \sin \theta = m \lambda \]

\[ \theta = \sin \theta \]

\[ m = 1 \]

\[ \lambda = \text{Red} \]