USEFUL CONSTANTS:

1. $e_0 = 8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2$
2. $k = 8.9875 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$
3. $\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2$
4. Magnitude of charge on an electron $e = 1.602 \times 10^{-19} \text{ C}$
5. Magnitude of charge on a proton $q_p = 1.602 \times 10^{-11} \text{ C}$
6. Mass of an electron $m_e = 9.11 \times 10^{-31} \text{ kg}$
7. Mass of a proton $m_p = 1.67 \times 10^{-27} \text{ kg}$

14. (6 points) A circular loop of wire of radius $R$ is placed in a uniform magnetic field $B$ and is then spun at a constant angular velocity $\omega$ about an axis through its diameter. If the axis of rotation is perpendicular to $B$, the magnetic flux through the loop varies with time given by the relation:

(A) $\pi R^2 B \cos(\omega t)$
(B) $\pi R^2 B \cos(\omega t)$
(C) $\pi R^2 B \cos(\omega t/2)$
(D) $\pi R^2 B \cos(\omega t/2)$

$\mathcal{E} = \int \mathbf{E} \cdot d\mathbf{A} = B \omega A \cos(\omega t)$
1. (8 points) A singly charged ion of mass \( m_1 \) is accelerated from rest by a potential difference \( V_a \). It is then deflected by a uniform magnetic field (perpendicular to the ion's velocity) into a semicircle of radius \( R_t \). Now a triply-charged ion of mass \( m_2 \) is accelerated through the same potential difference and is deflected by the same magnetic field into a semicircle of radius \( R_s = 2R_t \). The ratio of the ion masses \( m_2/m_1 \) is:

- \( A(\_\_) \quad 8 \)
- \( B(\_\_) \quad 12 \)
- \( C(\_\_) \quad 18 \)
- \( D(\_\_) \quad 24 \)
- \( E(\_\_) \quad 48 \)

<table>
<thead>
<tr>
<th>Solution</th>
<th>Option</th>
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<tbody>
<tr>
<td>( R_t = \frac{m_1 v_t}{q_1} )</td>
<td>( E )</td>
</tr>
<tr>
<td>( \frac{R_t}{R_s} = \frac{q_1}{q_2} )</td>
<td>( E )</td>
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\[ \frac{R_t}{R_s} = \frac{1}{2} \frac{q_1}{q_2} \]

\[ \frac{m_2}{m_1} = \frac{q_2}{q_1} = \frac{2}{1} = 2 \]

2. (8 points) Two parallel conductors, separated by a distance \( s = 30 \text{ cm} \), carry currents in the same directions. If \( I_1 = 2.0 \text{ A} \) and \( I_2 = 7.5 \text{ A} \). The force per unit length exerted on each conductor by the other is:

- \( A(\_\_) \quad 4.4 \times 10^{-4} \text{ N/m attractive} \)
- \( B(\_\_) \quad 4.4 \times 10^{-4} \text{ N/m repulsive} \)
- \( C(\_\_) \quad 1.0 \times 10^{-4} \text{ N/m attractive} \)
- \( D(\_\_) \quad 1.0 \times 10^{-4} \text{ N/m repulsive} \)
- \( E(\_\_) \quad 7.2 \times 10^{-4} \text{ N/m repulsive} \)

\[ \mathbf{F} = \int \mathbf{F} \cdot d\mathbf{r} = 0 \Rightarrow F = \frac{d\mathbf{F}}{dx} \]

\[ \mathbf{F} = \frac{\mu_0 I_1 I_2}{2\pi s} \mathbf{e}_x \]

3. (6 points) A small airplane with a wing span of 15 m is flying due north at a speed of 100 m/s over a region where the vertical component of the Earth's magnetic field is 1.2 \( \mu \text{T} \). The potential difference developed between the wing tips is:

- \( A(\_\_) \quad 7.44 \text{ mV} \)
- \( B(\_\_) \quad 1.05 \text{ mV} \)
- \( C(\_\_) \quad 0.23 \text{ V} \)
- \( D(\_\_) \quad 0.24 \text{ V} \)
- \( E(\_\_) \quad 1.60 \text{ V} \)

\[ \mathbf{E} = \frac{\mathbf{E}_z}{x} = \frac{\mathbf{E}_x}{x} = \frac{\mathbf{E}_y}{x} \]

\[ \mathbf{E} = \mathbf{V} \cdot \mathbf{B} = \mathbf{V} \cdot (9 \times 10^{-2} \mathbf{z}) \]

\[ \mathbf{E} = \mathbf{V} \cdot \mathbf{B} = \mathbf{V} \cdot (9 \times 10^{-2} \mathbf{z}) \]

\[ V = \frac{\mathbf{E}_y}{\mathbf{B}_y} = \frac{9 \times 10^{-2} \mathbf{z}}{9 \times 10^{-2} \mathbf{z}} \]

\[ V = 10 \times 10^{-2} \text{ V} \]
4. (6 points) The segment of wire in the figure below carries a current of \( I = 20.0 \, \text{A} \) in the direction shown by the arrow. The radius of the arc is \( R = 5.0 \, \text{cm} \). The magnetic field at point \( A \) is:

\[ B = \frac{\mu_0 I \sin \phi}{2 \pi R} \]

(A) \( 2 \times 10^{-8} \, \text{T out of the page} \)

(B) \( 2 \times 10^{-8} \, \text{T into the page} \)

(C) \( 2 \pi \times 10^{-8} \, \text{T in the plane of the page} \)

(D) \( 4 \pi \times 10^{-8} \, \text{T out of the page} \)

(E) \( 4 \pi \times 10^{-8} \, \text{T into the page} \)

5. (8 points) A circular loop of wire, of radius \( R = 50 \, \text{cm} \), with two turns lies in a plane perpendicular to a uniform magnetic field of magnitude \( B = 0.40 \, \text{T} \). If in the 0.10 s the wire is reshaped into a square with four turns, but remains in the same plane, what is the magnitude of the average induced emf (in Volts) in the wire during this time?

(A) \( 2 \pi [1 - \pi/16] \)

(B) \( 2 \pi [1 - \pi/4] \)

(C) \( 2 \pi [1 - \pi/8] \)

(D) \( 4 \pi [1 - \pi/8] \)

(E) \( 4 \pi [1 - \pi/8] \)

\[ \text{emf} = \frac{N \Phi}{\Delta t} \]

\( N = 2 \, \text{turns} \)

\( \Phi = \frac{B A^2}{2} \)

\( A = R^2 \)

\( \Phi = \frac{B R^2}{2} \)

\[ \text{emf} = \frac{2 \pi R^2}{2} \]

\[ = \frac{2 \pi R^2}{2} \left( 1 - \frac{3}{4} \right) \]
6. (8 points) A long solenoid, of radius \( R \), with a number of turns per meter carries a current which varies with time as \( I = I_0 \sin(\omega t) \). Inside the solenoid and coaxial with it is a loop that has a radius \( r < R \) and consists of a total of \( n \) turns of fine wire. The emf induced in the loop is given by:

\[
\begin{align*}
\text{(A)} & \quad -\pi R^2 n N_0 I_0 \sin(\omega t) \\
\text{(B)} & \quad -\pi r^2 n N_0 I_0 \sin(\omega t) \\
\text{(C)} & \quad \pi r^2 n N_0 I_0 \sin(\omega t) \\
\text{(D)} & \quad \pi R^2 n N_0 I_0 \sin(\omega t) \\
\text{(E)} & \quad \pi r^2 n N_0 I_0 \sin(\omega t)
\end{align*}
\]

\[B = \mu_0 n I_0 \sin(\omega t)\]

\[\varepsilon = -N \frac{d\Phi}{dt} = -N \mu_0 n I_0 \omega \cos(\omega t) \sin^2(\omega t)\]

7. (5 points) A 6-V battery is used to charge a 50-\( \mu \)F capacitor. The capacitor is then discharged through a 0.34 mH inductor. The maximum current in the circuit is:

\[
\begin{align*}
\text{(A)} & \quad 2.3 \text{ A} \\
\text{(B)} & \quad 2.8 \text{ A} \\
\text{(C)} & \quad 4.2 \text{ A} \\
\text{(D)} & \quad 5.1 \text{ A} \\
\text{(E)} & \quad 7.3 \text{ A}
\end{align*}
\]

\[\frac{L}{C} = \frac{100 \text{ mH}}{333 \text{ F}} = 0.3 \text{ s}^2\]

\[i_{\text{max}} = \frac{3 \text{ A}}{0.3} = 10 \text{ A}\]

8. (5 points) At \( t = 0 \), a source of emf with 500 V, applied to a coil that has an inductance of 0.80 H and a resistance of 30 ohms. The energy stored in the magnetic field when the current reaches half its maximum value is:

\[
\begin{align*}
\text{(A)} & \quad 14.7 \text{ J} \\
\text{(B)} & \quad 27.8 \text{ J} \\
\text{(C)} & \quad 38.2 \text{ J} \\
\text{(D)} & \quad 45.1 \text{ J} \\
\text{(E)} & \quad 64.8 \text{ J}
\end{align*}
\]

\[E_{\text{store}} = \frac{1}{2} L i^2 = \frac{1}{2} (0.8 \text{ H}) (0.5 \text{ A})^2 = 0.1\text{ J}\]
9. (8 points) A sinusoidal voltage \( V(t) = (40 \text{ V}) \sin(100t) \) is applied to a series LCR circuit with \( L = 160 \text{ mH} \), \( C = 99 \mu \text{F} \), and \( R = 68 \Omega \). The phase angle \( \phi \) between the current and the voltage is:

\[
\phi = \tan^{-1}\left( \frac{X_C - X_L}{R} \right) = \tan^{-1}\left( \frac{\omega L - \frac{1}{\omega C}}{R} \right)
\]

\[
\tan^{-1}\left( \frac{\omega L - \frac{1}{\omega C}}{R} \right) = \tan^{-1}\left( \frac{100 \times 10^{-3} \text{ H}}{68} \right) = -57.3^\circ
\]

10. (6 points) An RLC circuit is used in a radio to tune into an FM station broadcasting at 99.7 MHz. The resistance in the circuit is 12 \( \Omega \) and the inductance is 1.40 \( \mu \text{H} \). What value of capacitance should be used?

\[
\begin{align*}
(A) & \quad 1.15 \text{ pF} \\
(B) & \quad 1.62 \text{ pF} \\
(C) & \quad 1.26 \text{ pF} \\
(D) & \quad 2.29 \text{ pF} \\
(E) & \quad 5.32 \text{ pF}
\end{align*}
\]

\[
\omega^2 = \left( \frac{1}{LC} \right)^2 = \left( \frac{1}{(2\pi f)^2} \right) = \left( \frac{1}{(197 \times 10^6 \text{ Hz})^2} \right) = \left( \frac{1}{(6.1 \times 10^{-14} \text{ H})} \right)
\]

11. (8 points) A long straight conducting wire of diameter 4.0 mm carries a current of 20 A. Considering that the current is uniformly distributed over the cylindrical cross-section of the wire, calculate the magnetic field at a point 1.0 mm from the axis of the wire.

\[
\begin{align*}
(A) & \quad 0.40 \text{ mT} \\
(B) & \quad 1.0 \text{ mT} \\
(C) & \quad 0.50 \text{ T} \\
(D) & \quad 2.00 \text{ T} \\
(E) & \quad \text{None of the above}
\end{align*}
\]

\[
\begin{align*}
B &= \frac{\mu_0}{2\pi} \int_{-r}^{r} B_0 \cdot \frac{r}{r^2} \, dr \\
&= \frac{\mu_0 B_0 r}{2\pi} \int_{-r}^{r} \frac{1}{r^2} \, dr \\
&= \frac{\mu_0 B_0 r}{2\pi} \frac{1}{r} \\
&= \frac{10^{-6} \text{T} \cdot \text{m}^2}{\text{A}} \times \frac{2\pi r}{r} \\
&= 10^{-3} \text{T}
\end{align*}
\]
12. (8 points) Two straight, very long, parallel conductors carry currents $I_1$ and $I_2$ in the directions as shown in the figure below. If the magnetic field at point $C$ is to be zero, the ratio of the currents $I_1/I_2$ must be:

$$\frac{I_1}{I_2} = \frac{B}{2d}.$$

(A) 1.5
(B) 2.0
(C) 2.5
(D) 3.0
(E) At C, magnetic field can not be zero.

$\Rightarrow \frac{I_1}{I_0} = \frac{1}{2} = 0.5$

13. (8 points) An 800-eV electron traveling along the $+z$ axis enters a region of uniform magnetic field of magnitude 0.02 T. If the direction of the magnetic field is along $-z$, determine the magnitude and direction of the electric field necessary to keep the electron moving along its original direction.

(A) $3.36 \times 10^9$ V/m
(B) $-3.36 \times 10^9$ V/m
(C) $-3.36 \times 10^9$ V/m
(D) $3.36 \times 10^9$ V/m
(E) $7.72 \times 10^9$ V/m

$$E = \frac{m \omega \gamma}{e} = \frac{1.67 \times 10^{-31} \text{kg}}{1.6 \times 10^{-19} \text{C}} = 5.35 \times 10^5 \text{ V/m}.$$