Physics 220 – Final Exam

December 17 1999

This exam consists of 24 problems on 14 pages. Please check that you have them all.

All of the formulas that you will need are given below. You may also use a calculator.

\[
\sin \theta = \frac{y}{r} \quad \cos \theta = \frac{x}{r} \quad \tan \theta = \frac{y}{x}
\]

average speed = \( \frac{\text{distance traveled}}{\text{time}} \)

\( g = 9.8 \, \text{m/s}^2 \)

average velocity = \( \bar{v} = \frac{\text{displacement}}{\text{time}} \)

instantaneous velocity = slope of position versus time
instantaneous acceleration = slope of velocity versus time

For constant acceleration:

\[
x = x_0 + v_0 t + \frac{1}{2} a t^2
\]

\( v = v_0 + a t \quad v^2 = v_0^2 + 2a(x - x_0) \)

\[
F = m \ddot{a} \quad F_{\text{max}} = \mu_s N \quad (\text{static friction}) \quad F_{\text{friction}} = \mu_k N \quad (\text{sliding friction})
\]

\[
F_{\text{gravity}} = \frac{GMm}{r^2} \quad G = 6.67 \times 10^{-11} \, \text{Nm}^2/\text{kg}^2
\]

\[
\alpha_c = \frac{v^2}{r} \quad KE = \frac{1}{2} m v^2 \quad W = F \cdot d \cos \theta \quad PE_{\text{gravity}} = mgh
\]

power = work/\( \Delta t \) \quad \( \dot{p} = m \ddot{v} \quad \Delta p = \text{impulse} = F \Delta t \)

\[
F_{\text{spring}} = -kx \quad PE_{\text{spring}} = \frac{1}{2} k x^2 \quad x = A \sin(\omega t) \quad v = A \omega \cos(\omega t)
\]

\[
\omega = \sqrt{k/m} \quad \omega = \sqrt{g/L} \quad \omega = 2\pi f \quad f = 1/T
\]

For constant angular acceleration:

\[
\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 \quad \omega = \omega_0 + \alpha t \quad \omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)
\]

\[ KE = I\omega^2/2 \quad L = I\omega \quad \tau = I\alpha \quad \theta = \theta_0 + \omega_0 t \quad \omega = \omega_0 + \alpha t \quad \omega = \alpha t \quad \omega = \theta/t \]

The next page contains more formulas.
Pressure = Force/area \quad P_{\text{atm}} = 1.0 \times 10^5 \text{ Pa.} \quad A_1v_1 = A_2v_2 \\

P_1 + \rho gh_1 + \rho v_1^2/2 = P_2 + \rho gh_2 + \rho v_2^2/2 \\

Archimedes principle: buoyant force = weight of fluid displaced \\

\Delta L/L = \alpha \Delta T \\

PV = nRT \quad U_1 = 3nRT/2 \\

Avogadro's number = 6.02 \times 10^{23} \text{ particles per mole} \quad R = 8.31 \text{ J/mole - K} \\

KE_{\text{av}} = 3kT/2 \quad k = 1.4 \times 10^{-23} \text{ J/K} \quad \Delta U = Q - W \\

W = \text{area under } P - V \text{ curve} \\

v = f\lambda \quad T = 1/f \quad y = A\sin(2\pi ft - 2\pi x/\lambda) \quad y = A\cos(2\pi ft - 2\pi x/\lambda) \\

f_{\text{obs}} = f_{\text{source}}/(1 \pm v_a/v)
1. A car is initially traveling along a horizontal highway at 22 m/s. The driver then applies the brakes, and comes to a stop in 35 m. What was the magnitude of the acceleration of the car? Assume that the acceleration was constant during the braking process.

(a) 0.63 m/s²
(b) 0.31 m/s²
(c) 14 m/s²
(d) 6.9 m/s²
(e) 1.6 m/s²

\[
\sqrt{v^2} = v_0^2 + 2a(x-x_0)
\]
\[
0 = 22^2 + 2a(35)
\]
\[
a = -\frac{22^2}{2(35)} = -6.9 \text{ m/s}^2
\]

2. A girl of mass 65 kg skis down an icy slope for which friction is not negligible. She starts from rest at the top, which is 50 m above the finish line. If her speed at the bottom is 25 m/s, find the work done by friction.

\[
W_{\text{fric}} + KE_f + PE_f = KE_i + PE_i
\]
\[
W_{\text{fric}} = KE_f - PE_f
\]
\[
= \frac{1}{2}mv_f^2 - mgh
\]

(a) \(-3.2 \times 10^4\) J
(b) \(-1.2 \times 10^4\) J
(c) \(-2.0 \times 10^4\) J
(d) \(-5.2 \times 10^4\) J
(e) \(-4.1 \times 10^4\) J

\[
W_{\text{fric}} = \frac{1}{2}(65)(25)^2 - 65(9.8)(50)
\]
\[
= -1.16 \times 10^4
\]
3. A piston containing 7.0 moles of an ideal gas is initially at a temperature of 350 K with a gas pressure of $3.5 \times 10^6$ Pa. If 20,000 J of work is done on the gas and no heat is added, find the final temperature of the gas.

(a) 690 degrees K
(b) 368 degrees K
(c) 273 degrees K
(d) 580 degrees K
(e) 780 degrees K

\[ \Delta U = Q - W \]

\[ \frac{3}{2} nR (T_f - T_i) = 0 - (-20,000) \]

\[ 20 \times 10^3 \left( \frac{3}{5} nR \right) = 2 \times 10^4 \]

\[ T_f - T_i = \frac{2 \times 10^4}{2 \times 10^3} \]

\[ T_f - T_i = 2.29 \]

\[ T_f = 350 + 2.29 = 580 \text{ K} \]

4. A balloon containing oxygen molecules has a volume of 2.5 m$^3$ at a pressure of $4.0 \times 10^5$ Pa and a temperature of 400 K. Find the number of molecules in the balloon.

(a) $1.8 \times 10^{23}$
(b) $4.5 \times 10^{20}$
(c) $6.0 \times 10^{23}$
(d) $1.5 \times 10^{24}$
(e) $7.3 \times 10^{22}$

\[ PV = nRT \]

\[ n = \frac{PV}{RT} = \frac{4.0 \times 10^5 \times (2.5)}{8.31 \times 400} = 0.301 \text{ mol} \]

\[ \# \text{ of molecules} = n \times N_A = (0.301) \times 6.02 \times 10^{23} = 1.8 \times 10^{23} \]
5. Your body cools when you sweat. Which of the following statements explains why this happens?

(a) Because of the heat capacity of the sweat.
(b) Because the thermal expansion coefficient of sweat (water) is negative.
(c) Because of the latent heat of fusion.
(d) Because energy is conserved.
(e) Because of the latent heat of evaporation.

6. The figure below shows a snapshot of a wave on a string. Which interval is equal to the wavelength of this wave?

![Diagram of a wave on a string]

(a) A
(b) B
(c) C
(d) D
(e) E

Wavelength = repeat distance
7. Find the average speed of an oxygen molecule in the atmosphere. Assume that the atmosphere can be treated as an ideal gas (this is actually a good approximation) at a temperature of 300 K. The molecular mass of oxygen is 32 g (not kg).

\[
\frac{1}{2} \text{mt} \text{v}^2 = \frac{3}{2} k T
\]

\[
\text{v}_n = \sqrt{\frac{3 k T}{m}} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 300}{0.032}} = 490 \text{ m/s}
\]

8. A box of valuable metal of volume of 2.5 m³ is sitting on the bottom of a lake. If the force required to raise the box to the surface is 1.5 x 10⁶ N, find the density of the box. \( \rho_{\text{metal}} = 1000 \text{ kg/m}^3 \).

\[ F = m g - F_g \]

\[ F_x = \rho_{\text{box}} V g - \rho_{\text{metal}} V g \]

\[ \rho_{\text{box}} = \frac{F V g + F g}{V} \]

\[ = \frac{1.5 \times 10^6}{2.5 \times 7.8} + 1000 = 7100 \text{ kg/m}^3 \]
9. A rubber ball of mass 0.65 kg is thrown with an initial speed of 15 m/s. It collides with a wall, and bounces back in the opposite direction at a speed of 11 m/s. What is the magnitude of the impulse imparted to the ball?

(a) 7.2 N·s
(b) 9.8 N·s
(c) 17 N·s
(d) 2.6 N·s
(e) zero

Impulse = Δp = mΔv

Δv = 0.65(11 - (-15))

= 0.65(26) = 16.9 N·s

10. A man stands on a plank as shown below. The mass of the plank is 25 kg, and the mass of the man is 70 kg. Find the maximum value of L at which the man can stand without having the plank tip. Assume that the mass of the plank is distributed uniformly.

3.5 m

F_L = 0

2.5 m

1.75

0.75

L

m_p g

(a) 0.27 m
(b) 2.1 m
(c) 0.75 m
(d) 0.20 m
(e) 1.0 m

\[ \sum T = 0 \]

Compute the torque around point A

\[ m_p \cdot g \cdot 0.75 - m_u \cdot g \cdot L = 0 \]

\[ L = \frac{m_p \cdot (0.75)}{m_u} = \frac{25 \cdot (0.75)}{70} = 0.27 m \]
11. Your local water company delivers water to your house at a pressure such that when you open a faucet, the water comes out at a “pleasing” speed (i.e., not too fast and not too slow). If the water in your shower comes out at a speed of 10 m/s, what is the absolute pressure of the water in the pipe leading to the shower? Assume that the diameter of the shower head is very small. \( \rho_{\text{water}} = 1000 \text{ kg/m}^3; \ \rho_{\text{air}} = 1.3 \text{ kg/m}^3; \ \) \( P_{\text{atm}} = 1.0 \times 10^6 \text{ Pa} \)

\[
\begin{align*}
P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 &= P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2 \\
&= P_{\text{atm}}
\end{align*}
\]

(a) 2.0 \times 10^5 \text{ Pa}
(b) 4.5 \times 10^5 \text{ Pa}
(c) 1.0 \times 10^5 \text{ Pa}
(d) 0.5 \times 10^5 \text{ Pa}
(e) 1.5 \times 10^5 \text{ Pa}

\[
\begin{align*}
P_1 &= P_{\text{atm}} + \frac{1}{2} \rho (10)^2 \\
&= 1.0 \times 10^5 + \frac{1}{2} (1000) (10)^2 \\
&= 1.5 \times 10^5 \text{ Pa}
\end{align*}
\]

12. The frequency corresponding to the note middle C on the piano is approximately 262 Hz. What is the wavelength of a sound wave in air with this frequency? (The speed of sound in air is 330 m/s.)

(a) 0.63 m
(b) 1.3 m
(c) 3.1 m
(d) 0.79 m
(e) 17 m

\[
\begin{align*}
\lambda &= \frac{v}{f} \\
&= \frac{330}{262} = 1.26 \text{ m}
\end{align*}
\]
13. You are the lucky recipient of a new skateboard for Christmas. This is a special frictionless model, designed for high velocity work. To try it out, you go to a hill in your backyard. You start from rest at the top of the hill, and coast to the bottom. What is your speed when you reach the bottom of the hill? Assume that the hill is 7.5 m tall, and that the kinetic energy of the wheels and all friction are negligible.

\[
KE_A + PE_A = KE_f + PE_f
\]

\[
\frac{1}{2}m_1v_A^2 + m_1gh_A = \frac{1}{2}m_fv_f^2 + m_fgh_f = m_fgh_f
\]

\[
h_f = 0
\]

\[
h_A = 7.5
\]

\[
\begin{align*}
(a) & \quad 12 \text{ m/s} \\
(b) & \quad 150 \text{ m/s} \\
(c) & \quad 8.6 \text{ m/s} \\
(d) & \quad 36 \text{ m/s} \\
(e) & \quad 6.0 \text{ m/s}
\end{align*}
\]

\[
v_f = \sqrt{2gh_f} = \sqrt{2 \times 9.8 \times 7.5} = 12 \text{ m/s}
\]

14. Two masses are attached by a massless string which runs over a pulley as shown below. Find the acceleration of the mass on the right. Assume that the pulley is massless and frictionless.

\[
T - m_1g = m_1a
\]

\[
m_2g - T = m_2a
\]

\[
(m_2 - m_1)g = (m_1 + m_2)a
\]

\[
a = \frac{(m_2 - m_1)}{m_1 + m_2} \cdot \frac{g}{g} = \frac{4.5 - 3.0}{4.5 + 3.0} \cdot \frac{g}{g} = \frac{1.5}{7.5} \cdot \frac{g}{g} = 2.0 \text{ m/s}^2
\]
15. A sport utility vehicle of mass 2000 kg collides with a compact car of mass 800 kg. They lock bumpers and then roll off together at a final speed of 12 m/s. If the compact car was initially at rest. What was the speed of the sport utility vehicle just prior to the collision? Assume that all friction is negligible.

\[ m_1 \sqrt{v_{1x}} + m_2 \sqrt{v_{2x}} = (m_1 + m_2) \sqrt{v_f} \]

(a) 12 m/s  
(b) 17 m/s  
(c) 4.8 m/s  
(d) 30 m/s  
(e) 9.8 m/s

\[ \sqrt{v_{1x}} = \frac{m_1 + m_2}{m_1} \sqrt{v_f} \]

\[ = \frac{2000 + 800}{2000} \cdot 12 = \frac{2800}{2000} = 17 \text{ m/s} \]

16. Which of the following statements correctly describes longitudinal waves?

(a) The medium in which the wave travels moves very little.  
(b) The displacement of the medium is perpendicular to the direction of the wave.  
(c) The wave frequency decreases as the wave leaves its source.  
(d) The displacement of the medium is parallel to the direction of the wave.  
(e) The wave carries mass but no energy.
17. Guitar strings are often made of steel. If the temperature of a piece of steel is increased by 5.0°C, what happens to its length? Assume that the piece of steel is 0.75 m long, and that it is initially at room temperature. The coefficient of linear expansion of steel is \(1.2 \times 10^{-5}/°C\).

\[ \Delta L = \alpha L_0 \Delta T \]
\[ = 1.2 \times 10^{-5} (0.75) \times 5 \]
\[ = 4.5 \times 10^{-5} \text{m} \]

(a) The length increases by \(6.0 \times 10^{-5} \text{m}\)
(b) The length does not change
(c) The length increases by \(9.0 \times 10^{-6} \text{m}\)
(d) The length decreases by \(6.0 \times 10^{-5} \text{m}\)
(e) The length increases by \(4.5 \times 10^{-5} \text{m}\)

18. Which of the following statements concerning gasses and kinetic theory is correct?

(a) Ideal gasses can be cooled to absolute zero.
(b) The ideal gas law can be applied to any fluid, be it a liquid or a gas.
(c) Ideal gasses do not melt.
(d) The temperature of a gas is directly proportional to the kinetic energy of the gas molecules.
(e) The term "ideal" in the ideal gas law means that collisions between gas molecules are always elastic.
19. A watermelon is dropped from the top of a bridge into the lake below. If the distance from the bridge to the lake below is 25 m, and the mass of the watermelon is 2.5 kg, how long does it take the watermelon to reach the water? Assume that the initial velocity of the watermelon is zero.

\[ y = y_i + v_y t - \frac{1}{2} g t^2 \]

(a) 2.3 s
(b) 3.9 s
(c) 9.8 s
(d) 5.1 s
(e) 4.3 s

\[ 0 = 25 + 0 - \frac{1}{2} \times 9.8 \times t^2 \]

\[ t = \sqrt{\frac{2 \times 25}{9.8}} = 2.35 \]

20. A bullet is fired from a rifle. The bullet has an initial speed of 400 m/s, and is fired at an angle of 45° with respect to the horizontal. What is the speed of the bullet when it reaches its maximum height? Ignore air resistance.

\[ \text{at this point the velocity will be purely horizontal} \]

(a) 400 m/s
(b) 570 m/s
(c) 41 m/s
(d) 200 m/s
(e) 280 m/s

\[ V_x = V_x0 = 400 \cos 45° = 280 \text{ m/s} \]
21. Two boxes are stacked on a table as shown below. Everything is motionless. There is a normal force from the bottom block on the top block, and this force is \( N = m_1 g \) directed up. According to Newton's third law, for every force, there is a reaction force. What is the reaction force to \( N \), and what does it act on?

(a) \( m_1 g \) acting down on the table
(b) \( m_1 g \) acting on the earth
(c) \( (m_1 + m_2)g \) acting down on the table
(d) zero

\( \boxed{\text{(e) } m_2 g \text{ acting down on the bottom block}} \)

22. The internal energy of an ideal gas which is trapped in a piston increases by 3000 J during a thermodynamic process. If this process is adiabatic (which means that no heat is added to or removed from the system), how much work is done by the gas on its surroundings?

\[ \Delta U = Q - W \]

(a) zero
(b) -1500 J
(c) 6000 J

\( \boxed{\text{(d) } -3000 \text{ J}} \)
(e) +3000
23. A wave on a string is described by the equation \( y = 17 \cos(7.5t - 4.2x) \) where the units of distance are m and the units of time are s. What is the frequency of this wave?

(a) 2.1 Hz  
(b) 0.67 Hz  
(c) 1.2 Hz  
(d) 4.2 Hz  
(e) 0.24 Hz

\[
y = A \cos(2\pi ft - \frac{2\pi}{\lambda} x)
\]

\[
\begin{align*}
2\pi f &= 7.5 \\
\frac{7.5}{2\pi} &= 1.2 \text{ Hz}
\end{align*}
\]

24. 2.5 moles of \( H_2 \) gas are placed in a container of volume 1.5 m\(^3\) at a temperature of 300 K. Find the internal energy of the system. The molecular weight of hydrogen gas is 1.0 g.

(a) 750 J  
(b) 6200 J  
(c) zero  
(d) 9300 J  
(e) \( 6.2 \times 10^4 \) J

\[
U = \frac{3}{2} n RT = \frac{3}{2} (2.5)(8.31) \cdot 300
\]

\[
= 9300 \text{ J}
\]

The End