Physics 220 – Exam #3

November 13 2002

This exam consists of 12 problems on 7 pages. Please check that you have them all.

All of the formulas that you will need are given below. You may also use a calculator.

\[
\sin \theta = \frac{y}{r} \quad \cos \theta = \frac{x}{r} \quad \tan \theta = \frac{y}{x}
\]

Average speed = \( \text{distance traveled} \div \text{time} \) \[ y = 9.8 \text{ m/s}^2 \]

Average velocity = \( \frac{\text{displacement}}{\text{time}} \)

Instantaneous velocity = slope of position versus time

Instantaneous acceleration = slope of velocity versus time

For constant acceleration:

\[ x = x_0 + v_0 t + \frac{1}{2} a t^2 \quad v = v_0 + at \quad v^2 = v_0^2 + 2a(x - x_0) \]

\[ F = m\ddot{x} \quad F_{\text{friction}} = \mu_s N \quad F_{\text{friction}} = \mu_k N \]

\[ F_{\text{gravity}} = \frac{m \text{ms}}{2} \quad G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 \]

\[ a_c = \frac{v^2}{r} \quad KE = \frac{1}{2}mv^2 \quad W = Fd \cos \theta \quad PE_{\text{gravity}} = mgh \]

Power = work/\( \Delta t \) \[ \dot{p} = m\ddot{x} \quad \Delta p = \text{impulse} = F \Delta t \]

\[ F_{\text{spring}} = -kx \quad PE_{\text{spring}} = \frac{1}{2}kx^2 \quad x = A \sin(\omega t) \quad v = A \omega \cos(\omega t) \]

\[ \omega = \sqrt{\frac{k}{m}} \quad \omega = \sqrt{\frac{g}{L}} \quad \omega = 2\pi f \quad f = 1/T \]

For constant angular acceleration:

\[ \theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 \quad \omega = \omega_0 + \alpha t \quad \omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0) \]

\[ KE = \frac{1}{2}I\omega^2 \quad L = I\omega \quad \tau = I\alpha \quad \theta = \frac{s}{r} \quad \omega = v/r \quad \alpha = a/r \quad \omega = \theta/t \]

Pressure = Force/area \[ A_1v_1 = A_2v_2 \quad P_1 + \rho g h_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2} \rho v_2^2 \]

Archimedes principle: buoyant force = weight of fluid displaced
1. A compact disk of radius 0.080 m is initially at rest. The CD player is then turned on and the CD accelerates uniformly to an angular speed of 8.5 radians/s after 5.0 s. How many revolutions does the CD make during this time?

(a) 21  
(b) 3.4  
(c) 6.7  
(d) 43  
(e) 8.5

\[ \omega_f - \omega_i = \frac{8.5 \text{ rad}}{5.0 \text{ s}} \]

\[ \omega = \omega_i + \frac{1}{2} \alpha t^2 = \frac{1}{2} \alpha (5.0 \text{ s})^2 = 21.25 \text{ rad} \]

\[ \text{He} = \frac{\omega_f}{\omega} = \frac{6.7}{21.25} = 0.31 \text{ rad} \]

2. A hockey puck (mass 0.35 kg) slides on a frictionless semi-circular track as shown. If the puck starts from rest at point A (where it is shown in the figure), what is the normal force between the track and the puck when the puck reaches point B (the bottom of the track)? The radius of the track is 4.5 m.

![Diagram of a hockey puck on a frictionless semi-circular track.]

(a) 7.0 N  
(b) 15 N  
(c) 3.5 N  
(d) 2.5 N  
(e) 10 N

\[ mgh = \frac{1}{2} m v^2 \]

\[ m v^2 = 2 mgh \]

\[ m v^2 = 2 m g \frac{h}{R} \]

\[ N - m g = m \frac{v^2}{R} \]

\[ N = m g + m \frac{v^2}{R} \]

\[ = 3 m g + 2 m g \frac{h}{R} \]

\[ = 3 m g + 2 m g \frac{4.5}{4.5} = 10 N \]
3. A particle attached to a spring with spring constant $k = 400 \text{ N/m}$ is undergoing simple harmonic motion as it moves on a frictionless horizontal surface. The position of the particle is given by the equation $x = 0.12 \cos(\omega t)$ (in units of meters) with $\omega = 48 \text{ rad/s}$. Find the potential energy at $t = 1.3 \text{ s}$.

\[
PE = \frac{1}{2} k x^2 = \frac{1}{2} (400) \left[ 0.12 \cos \left( 48 \cdot 1.3 \right) \right]^2
\]

\[
= \frac{200}{0.0119}
\]

\[
= 2.4 \text{ J}
\]

4. A block slides down an inclined plane as shown below. There is friction between the block and the plane. If the acceleration of the block is $2.5 \text{ m/s}^2$, find the coefficient of kinetic friction between the block and the plane.

\[
m = 8.5 \text{ kg}
\]

\[
\mu \text{ N} = m \text{ g} \sin \theta
\]

\[
m \text{ g} \sin \theta - \mu \text{ N} = m \text{ a}
\]

\[
mg \sin \theta - \mu mg \cos \theta = ma
\]

\[
m \text{ g} \cos \theta = g \sin \theta - a
\]

\[
\mu = \frac{g \sin \theta - a}{g \cos \theta} = \frac{4.14 - 2.5}{5.88} = \frac{1.64}{5.88} = 0.28
\]

\[
= 0.18
\]
5. A sphere rolls down a ramp from an initial height of 2.5 m. Find the speed of the sphere when it reaches the bottom of the ramp. \( I(sphere) = \frac{2}{5}mR^2 \).

(a) 5.9 m/s
(b) 12 m/s
(c) 7.0 m/s
(d) 40 m/s
(e) 4.9 m/s

\[ KE_A + PE_A = KE_f + PE_f \]
\[ 0 + mgh = \frac{1}{2}mN^2 + \frac{1}{2}I\omega^2 \]
\[ \omega = \sqrt{\frac{2}{5}} \]
\[ \frac{1}{2}I\omega^2 = \frac{1}{2} \cdot \frac{2}{5}mR^2 \cdot \left( \frac{\sqrt{2}}{R} \right)^2 \]
\[ = \frac{1}{5}mN^2 \]
\[ mgh = \frac{1}{2}mN^2 + \frac{1}{5}mN^2 = \frac{7}{10}mN^2 \]
\[ N = \sqrt{\frac{10}{7}gh} = 5.9 \text{ m/s} \]

6. A block of mass 22 kg is attached to a spring. The block is pulled to one side a distance 0.24 m with a force of 60 N. The block is then released from rest, and undergoes simple harmonic motion. What is period of the motion? Assume that there is no friction between the block and the surface on which it moves.

(a) 3.4 s
(b) 0.30 s
(c) 1.9 s
(d) 250 s
(e) 4.8 s

\[ F = kx \]
\[ k = \frac{|F|}{x} = \frac{60}{0.24} = 250 \text{ N/m} \]
\[ W = \sqrt{\frac{k}{m}} = 2\pi \]
\[ T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{22}{250}} = 1.95 \]
7. One end of a plank of length 7.5 m and mass 55 kg rests on a support while a cable is attached to the other end. The mass of the plank is distributed uniformly. There is a person of mass 80 kg standing 2.5 m from one end of the plank. Find the tension in the cable.

\[ \sum T = 0 = -T \cdot L + m_1 \frac{L}{2} + m_2 g (L - 2.5) \]

\[ T = \frac{m_1 g}{2} + m_2 g \frac{5}{7.5} \]

\[ = 269.5 + 522.7 = 792 \text{ N} \]

8. A 2.5 kg block is dropped straight down onto a vertical spring with \( k = 500 \text{ N/m} \). The block sticks to the spring, and the spring compresses 0.22 m before coming to rest. What was the speed of the block just before it hit the spring?

\[ \frac{1}{2} m v^2 = \frac{1}{2} k x^2 - m g x \]

\[ v = \sqrt{\frac{k x^2 - 2 g x}{m}} \]

\[ = \sqrt{\frac{500 (0.22)^2 - 2 (9.8) x}{2.5}} \]

\[ = \sqrt{9.68 - x^2} \]

\[ = 2.3 \text{ m/s} \]
9. One end of a uniform pole of length 3.0 m is held by the corner between a wall and the floor, and is free to rotate, while the other end is attached to a spring as shown. The pole is at rest, the angle between the pole and the floor is 45°, and the spring is horizontal. If the mass of the pole is 15 kg, and the spring constant of the spring is 300 N/m, find amount that the spring is stretched.

\[ \frac{2}{3} \cdot 2 = 0 \]
\[ m \cdot g \cdot \frac{L}{2} \cdot \cos 45^\circ = k \cdot x \cdot L \cdot \sin 45^\circ \]

(a) 3.0 m
(b) 0.050 m
(c) 0.49 m
(d) 150 m
(e) 0.25 m

10. You are tired of suburbia and decide to live in a cave. Your cave is carefully crafted so that air flows past the two entrances at different speeds, and this leads to the circulation of fresh air through the cave. In the cave shown below, the air speeds are \( V_A = 7.3 \text{ m/s} \) and \( V_B = 5.4 \text{ m/s} \). Find the difference in air pressure between the two openings to your cave, and the direction in which the air circulates. Assume that the two openings are at the same height. \( \rho_{air} = 1.3 \text{ kg/m}^3; P_{air} = 1.0 \times 10^5 \text{ Pa} \)

\[ 7.3 \text{ m/s} \]

(a) 16 Pa, counterclockwise circulation through the cave
(b) 16 Pa, clockwise circulation
(c) 1.2 Pa, counterclockwise circulation
(d) 34 Pa, clockwise circulation
(e) 34 Pa, counterclockwise circulation

\[ P_A + \frac{1}{2} \rho V_A^2 = \frac{1}{2} \rho V_B^2 + \Delta P \]

\[ \Delta P = \frac{1}{2} \rho \left( V_A^2 - V_B^2 \right) \]

\[ \Delta P = \frac{1}{2} \rho \left( 7.3^2 - 5.4^2 \right) \]

\[ = \frac{1}{2} \rho \left( 53.29 - 29.16 \right) \]

\[ = 16 \text{ Pa} \]
11. You are an adventurer who wants to fly around the world in a helium filled balloon. You want the balloon to be able to lift you plus your supplies (a total mass of 400 kg), along with the weight of the balloon itself. What is the minimum volume that the balloon must have to get off the ground? \( (\rho_{\text{air}} = 1.3 \text{ kg/m}^3; \rho_{\text{helium}} = 0.18 \text{ kg/m}^3; \ P_{\text{atm}} = 1.0 \times 10^5 \text{ Pa})\) Hints: Don't forget the mass of the helium in the balloon. You can ignore the mass of the fabric of the balloon.

- (a) 2200 m³
- (b) 160 m³
- \( \boxed{\text{(c) 360 m³}} \)
- (d) 310 m³
- (e) 530 m³

\[ F_B = m_{\text{total}} g \]

\[ \rho_{\text{air}} V g = (m_{\text{supply}} + m_{\text{He}}) g \]

\[ m_{\text{supply}} + m_{\text{He}} = (m_s + \rho_{\text{He}} V) g \]

\[ V \left( \rho_{\text{air}} - \rho_{\text{He}} \right) = m_s \frac{400}{1.3 - 0.18} = 360 \text{ m}^3 \]

12. You are in charge of a submarine mission to explore a deep ocean region that is 2300 m below the surface of the ocean. Your submarine is a cube that is 6.0 m on a side. Before starting, you want to be sure that a window of area 2.5 m² on the top surface of the submarine will not collapse under the extreme pressure. Calculate the total force on the window from the water pressure. \( (\rho_{\text{water}} = 1000 \text{ kg/m}^3; \ P_{\text{atm}} = 1.0 \times 10^5 \text{ Pa}) \)

- (a) \( 2.3 \times 10^7 \text{ N} \)
- (b) \( 3.3 \times 10^8 \text{ N} \)
- (c) \( 9.4 \times 10^7 \text{ N} \)
- \( \boxed{\text{(d) 5.7 \times 10^7 \text{ N}}} \)
- (e) \( 4.9 \times 10^8 \text{ N} \)

\[ F = P_A \]

\[ P = P_0 - \rho gh \]

\[ = 1.0 \times 10^5 + 1000 (9.8)(2300) \]

\[ = 2.26 \times 10^7 \text{ N} \]

\[ F = P (2.5) = 5.7 \times 10^7 \text{ N} \]

The End