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Simulations using echo sequences to observe coherence in a cold Rydberg gas

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Abstract

We simulate the effect of special excitation pulses on a cold gas of atoms. First, a rotary echo sequence is used to examine the coherent nature of a frozen Rydberg gas. If collective excitation and de-excitation is present with little or no source of dephasing, after these pulses the system should be returned to a state with few excitations, and a strong echo signal should occur. We investigate systems that should display a perfect echo and systems where the interaction between atoms reduces the echo signal. A spin echo sequence is also used on a system of coherent hopping excitations, and we simulate how the strength of a spin echo signal is affected by thermal motion.

1. Introduction

Advancements in cooling and trapping have opened up new opportunities for investigating the properties of interacting many-body systems. In particular, the creation of frozen Rydberg gases have made it possible to study correlated groups of atoms [1, 2]. At these low temperatures the motion of the atoms can be neglected during the time scales of excitation, and the long-range interactions between atoms can be carefully studied.

One interesting consequence of the strong interaction between Rydberg atoms is the suppression of excitation known as the dipole blockade effect [3, 4]. While the reduction in excitation has experimentally been seen by several groups [4–10], it is of recent interest to measure the coherent collective behaviour of the groups of atoms that have been blocked from becoming excited [7, 11].

The dipole–dipole interaction between Rydberg atoms can also lead to a situation where two pairs of states ($|AB\rangle$, $|B'A'\rangle$) are moved into resonance with each other ($E_A+E_B \simeq E'_B+E'_A$) [1, 2, 12, 13]. In this case, the system will oscillate between the two states at a rate governed by the dipole–dipole interaction between them. With more than two atoms involved in the system, the states appear to coherently hop from atom to atom [14, 15]. If the atoms are placed into regular lattice sites then a direct observation of the coherent hopping can be detected [15], but in a random gas the coherent nature of the hopping is hidden.

In this paper, we simulated the effect of echo sequences on coherent Rydberg systems by using the many-body pseudoparticle wavefunction approach outlined in [16, 17] and the essential states model used in [14] to numerically solve the Schrödinger equation. In particular, we investigated the rotary echo of a strongly blocked Rydberg gas and the spin echo of a system of hopping excitations. The approach taken in [17] is particularly appropriate to use in the strong dipole blockade regime because we explicitly correlate groups of nearby atoms and take into account the spatial correlations between pseudoparticles that a simple mean-field model cannot. The hopping dynamics are well described by the essential states model.

2. Rotary echo of a dense Rydberg gas

When the system is especially dense, the correlations within a gas can become the dominant factor in the dynamics of the system. An example of this is the coherent Rydberg excitation of dense ultracold atoms [7]. In this case, the van der Walls interaction ($V(R) \propto 1/R^6$) between excited states actively suppresses the number of atoms able to be excited to Rydberg states, thus exhibiting a dipole blockade [3]. The minimum distance between excited particles, or blockade radius (R_b), is governed by the distance where $V(R)$ is comparable to the linewidth of excitation. Only one excited atom within the blockade region swept out by $R_b/2$ is allowed, and this single excitation is delocalized across all N_b atoms contained within this volume. The collective Rabi

oscillation rate Ω of this collection, or 'superatom' [18], of N_b atoms is given by $\Omega = \sqrt{N_b}\Omega_0$, where Ω_0 is the Rabi frequency of an isolated atom. In the experiment performed in [7], the dominant contribution to the linewidth of excitation is power broadening which is proportional to Ω. Now the minimum distance between Rydberg atoms is related to the linewidth by $R_b \propto (\sqrt{N_b}\Omega_0)^{-1/6}$, but N_b is determined by the local atomic density, ρ, and the size of the blockade region: $N_b \propto \rho R_b^3$. After substitution, it becomes clear that the size of the blockade region and N_b are related to the local density by $R_b \propto \rho^{-1/15}$ and $N_b \propto \rho^{4/5}$. Ultimately, the collective oscillation rate of a superatom is dependent on the local density by $\Omega \propto \rho^{2/5}$ [17]. In a typical magneto-optical trap (MOT), the density of the gas spans over several orders of magnitude; therefore, superatoms within the gas will oscillate over a wide range of frequencies. In fact, by using the MOT parameters in [7], most of the superatoms in the gas oscillate about ten times slower than those located near the peak density [17]. This inhomogeneity in density (and therefore collective oscillation frequency) makes it very difficult for experimental studies to directly measure the coherent nature of the system because the observable is an integration over the entire sample [11].

Early studies in the field of nuclear magnetic resonance physics had to overcome similar problems with inhomogeneities in magnetic fields which led to a wide range of Larmor precession frequencies and obscured the resonant absorption of the driving RF field [19, 20]. In 1950, Hahn demonstrated the effectiveness of a 'spin echo' sequence of pulses that was extremely effective in eliminating noise from the signal. In 1959, Solomon also demonstrated the successful use of a 'rotary echo' in doped water to overcome the effects of inhomogeneities in magnetic fields.

More recently, there was an experiment which used a rotary echo technique to prove the coherence of the excitation in a strongly blockaded ultracold gas [11]. The experimental setup in [11] trapped and cooled atoms down to 3.8 μK and excited them to $43S_{3/2}$ for up to 500 ns while keeping track of the total number of excitations in the gas. At such a low temperature and short excitation time the atoms are effectively motionless, so thermal motion can be disregarded as an outside source of decoherence. In a system of ultracold Rydberg atoms, a substantial source of inhomogeneity in the Hamiltonian is the variation in local density across the sample; the Gaussian shape of the density distribution in [11] certainly led to an inhomogeneity in Ω.

2.1. Rotary echo

In simplest terms a rotary echo sequence flips the sign of the excitation amplitude in the Hamiltonian after a certain time τ_p. The Hamiltonian describing the excitation of a dense ultracold gas using the many-body pseudoparticle approach is

$$\begin{aligned}\hat{H}(t) &= \sum_j \hat{H}_j^{(1)}(t) + \sum_{j<k} V_{jk}|n_j n_k\rangle\langle n_j n_k| \\ \hat{H}_j^{(1)}(t) &= -\left[\Delta\omega(t) + \varepsilon(t)\right]|n_j\rangle\langle n_j| \\ &\quad + \mathcal{F}(t)\frac{\Omega_0}{2}\sqrt{N_j}(|g_j\rangle\langle n_j| + |n_j\rangle\langle g_j|),\end{aligned} \tag{1}$$

where the number of atoms associated with pseudoparticle j is given by N_j, and V_{jk} is the interaction between two pseudoparticles j and k. One way the interaction V_{jk} can be calculated is by averaging over the interactions V between all of the pairs of associated atoms,

$$V_{jk} = \frac{1}{N_j N_k}\sum_{n\in j}\sum_{m\in k} V_{nm}(R_{nm}), \tag{2}$$

where R_{nm} is the distance between atoms n and m, which belong to pseudoparticles j and k respectively. Another way to calculate the interaction would be to simply use the positions of the pseudoparticles themselves. Now $V_{jk} = V(R_{jk})$. Clearly as the number of pseudoparticles is increased the two different methods will give the same interaction energies. We used the latter approach in our calculations because it converged faster with respect to random geometries as long as enough pseudoparticles were used. The detuning of the laser is $\Delta\omega(t)$, and $\varepsilon(t)$ is a mean-field energy shift due to excited atoms outside of the simulated volume. The kets $|g_j\rangle$ and $|n_j\rangle$ correspond to atom j being in the ground state and excited to the n manifold respectively, and $|n_j n_k\rangle$ is the state where atoms j and k are both excited. In [11], the rotary echo sequence was accomplished by using an RF field to flip the sign of Ω_0 after the time $\tau_p \leqslant \tau$, where τ is the total excitation time. In our simulations, we used the following to model the excitation profile:

$$\mathcal{F}(t) = \begin{cases} e^{-5(t-t_r)^2/t_r^2} & \text{for} \quad t \leqslant t_r \\ 1 & \text{for} \quad t_r < t \leqslant \tau_p - t_r \\ e^{-2(t-(\tau_p-t_r))^2/t_r^2} & \\ \quad - e^{-2(t-(\tau_p+t_r))^2/t_r^2} & \text{for} \quad \tau_p - t_r < t \leqslant \tau_p + t_r \\ -1 & \text{for} \quad \tau_p + t_r < t \leqslant \tau - t_r \\ -e^{-5(t-(\tau-t_r))^2/t_r^2} & \text{for} \quad \tau - t_r < t \leqslant \tau. \end{cases} \tag{3}$$

In equation (3), t_r is the ramping time for the laser. A Gaussian is used during ramp on time, between sign changes, and ramp down time as a smooth transition to avoid an instantaneous switch which could lead to unphysical, and therefore undesired high- frequency effects. Figure 1 illustrates an example of this type sequence.

When the system is sufficiently sparse, the energy shift due to the van der Waals interaction (V_{jk}) between atoms j and k becomes negligibly small compared to the width of the excitation amplitude, and the ground-state atoms are excited for a time τ_p and de-excited for a time $\tau - \tau_p$. This system of isolated atoms will be returned to zero excitations if $\tau_p = \tau/2$. A measurement of zero excitations is the perfect rotary echo case. Figure 2 is the result of simulating the excitation of a diffuse system of five atoms using a rotary echo sequence. The excitation amplitude Ω_0 was chosen so the maximum number would be excited at 500 ns. These echo simulations compare the number of excited atoms to the timing of the sign flip τ_p. As expected, when $\tau_p = \tau/2$ a perfect echo was recorded.

If the system is so dense that the van der Waals interactions are much greater than the excitation amplitude then the system is in the strong blockade limit. In the extreme case where all of the atoms are within a blockade radius the system has been reduced down to a single superatom. This single isolated

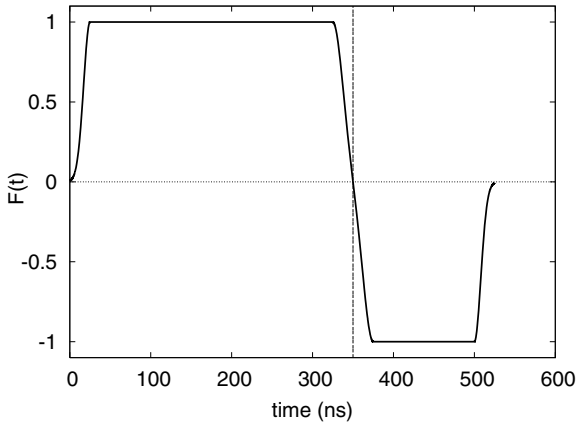

Figure 1. An illustration of a rotary echo sequence. The sign of the excitation amplitude $\mathcal{F}$ is smoothly flipped after time τ_p. In this case $\tau_p = 350$ ns, and is indicated by the dashed line.

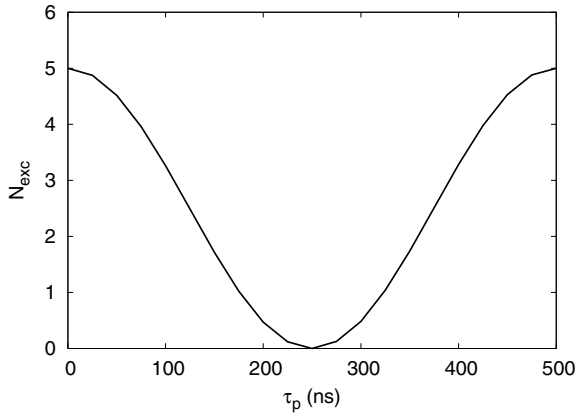

Figure 2. Number excited versus the timing of the sign change of the excitation amplitude. This echo signal is for five isolated atoms. Note that there are zero excitations when $\tau_p = 250$ ns, exactly half of the total excitation time.

superatom will also be excited and de-excited for the same amount of time, thus returning the system to a state of zero excitations. In figure 3, we plot the number excited versus τ_p for a perfectly blocked system of ten atoms (all ten atoms lie within a region defined by $R_b/2$). A perfect echo can be seen when $\tau_p = \tau/2$ and the maximum number excited is exactly 1.

A more interesting situation arises when the system is dense enough to create blockades, yet large numbers of excitations are allowed to occur. Now the energy shift due to the van der Waals interaction is comparable to the width of the excitation amplitude. In this case, the pseudoparticle approach used in [16, 17] is especially useful in describing the spatial correlations between pseudoparticles which represent groups of blockaded atoms. In this model, the van der Waals interaction is between pseudoparticles and not between individual atoms themselves, so the inhomogeneity in collective oscillations is explicitly included in the simulation. We simulated a rotary echo sequence of excitation for two different peak densities. In each case, we simulated the

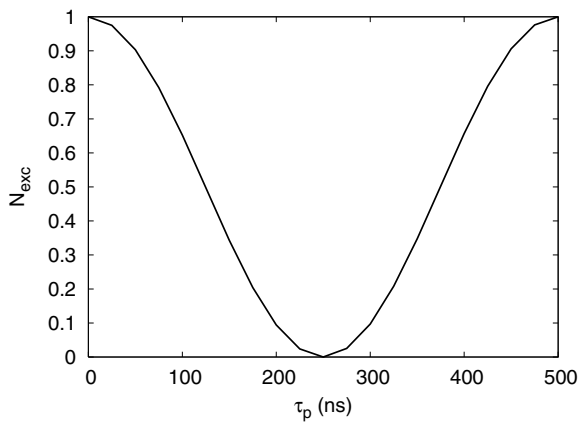

Figure 3. Number excited versus the timing of the sign change of the excitation amplitude. This echo signal is for ten atoms in the perfect blockade regime which means that the maximum number of excited atoms in the collection is exactly 1. Note the perfect echo signal at 250 ns.

fraction excited at specific density points and convolved the results over the Gaussian density distribution given in [7]

$$N_{\mathrm{exc}}(\tau) = 2\pi\sigma^2\Delta z \int P_e(\rho,\tau)\sqrt{\ln(\rho_0/\rho)}\,\mathrm{d}\rho, \qquad (4)$$

where $\sigma = 12\,\mu$m is the width of the excitation region in the radial direction, $\Delta z = 220\,\mu$m is the width in the axial direction, ρ_0 is the peak density and $P_e(\rho,\tau)$ is the fraction excited at a given fixed density, ρ, after an excitation time τ. The other parameters used in the following three simulations were chosen to match those used in [11]. Plotted in figure 4(a) is the echo signal as a function of τ_p for a peak density of 5×10^{12} cm^{-3}. While the echo signal is not perfect at $\tau_p = \tau/2$, it is still prevalent because the interaction between pseudoparticles is not as strong. When simulating at a higher peak density of 1.5×10^{13} cm^{-3}, the echo signal is weakened as in figure 4(b) by the dephasing caused by pseudoparticle interactions.

Interactions between pseudoparticles are a significant source of dephasing which prevents a perfect echo from occurring. This can be understood in the context of the Hamiltonian for the system. While the off-diagonal elements (given by the excitation amplitude) are reversed after τ_p, the sign of the diagonal van der Waals interaction elements remain unchanged, and the system does not perfectly evolve backwards in time. If it were possible to switch the sign of the van der Waals interaction between particles at the same time as the excitation amplitude then the sign of the entire Hamiltonian would be flipped and all sources of dephasing would have to be external such as thermal motion or ionization. Flipping the sign of the whole Hamiltonian is effectively the same as reversing the sign of t and perfectly running the system backwards in time. The lack of a perfect rotary echo signal would indicate outside sources of decoherence on the gas and coupling to external degrees of freedom. In figure 5, we simulated a system where the whole Hamiltonian gets switched in sign at τ_p in the same manner as equation (3). The perfect echo indicated in our simulations is consistent with the notion of running the system backwards in time. We

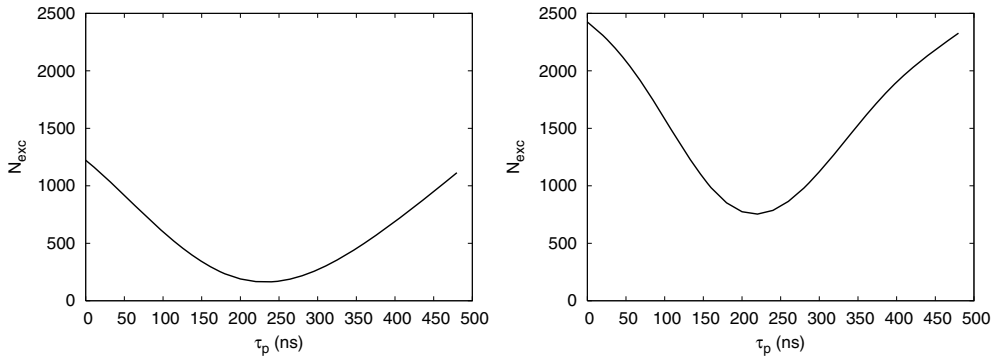

Figure 4. Number excited versus the timing of the sign change of the excitation amplitude. The echo signal for (a) peak density of $\rho = 5.0 \times 10^{12}$ cm^{-3} and (b) peak density $\rho = 1.5 \times 10^{13}$ cm^{-3}.

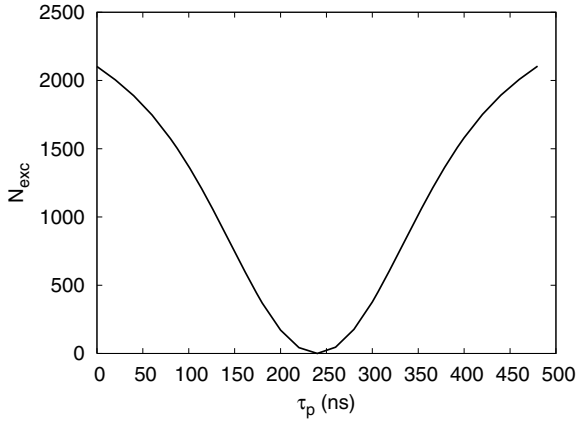

Figure 5. Number excited versus the timing of the sign change of the entire interaction Hamiltonian. This echo signal is for a high peak density of $\rho = 1.5 \times 10^{13}$ cm^{-3}.

believe that it might be possible to realize a time-reversible system of Rydberg atoms by using a small electric field to diabatically push the system to states that experience a dipole interaction of the same magnitude but opposite in sign compared to the field-free states. A promising interaction would involve taking advantage of the resonance near $n = 43$ for $d - d$ Rb atoms.

3. Spin echo for coherent Rydberg hoppers

In [14], a possible experiment to measure the coherent hopping of excitation between slightly irregular lattice sites was proposed. A key feature of this proposed experiment was the clear separation of regions which would allow for the spatial location of the hopper to be measured, but it would be impossible to measure the coherent nature of the hopping in an inhomogeneous gas in this manner. The amount of time it takes for an excitation to hop between atoms is inversely proportional to the dipole–dipole interaction energy between them, $t_{\rm hop} \propto R^3$ [14], so in an irregular gas of Rydberg atoms the excitations would be hopping at different rates and directions. Any measurement of hopping rates or locations would be hidden due to the random placement of Rydberg atoms. Unlike the previous section, all state labels that follow will refer to single atom states and not pseudoparticle states.

3.1. Spin echo

Like the rotary echo, the spin echo sequence was first used to overcome inhomogeneities in the magnetic field in NMR. Unlike the rotary echo, the spin echo is not simply a consequence of running the system forwards and backwards in time for the same duration. The spin echo works by making series of unitary transformations on the generalized Bloch sphere describing the quantum system [19]. In particular, we used the following sequence of pulses to simulate a spin echo: first the system is excited by a $\pi/2$ pulse (which takes the ground state $|g\rangle$ to a mix of the ground and excited state $(|g\rangle - \mathrm{i}|n\rangle)/\sqrt{2}$, and an excited state $|n\rangle$ to a mix of $|n\rangle \rightarrow (-\mathrm{i}|g\rangle + |n\rangle)/\sqrt{2}$), then the system is allowed to dephase for a time τ_1. Next, the system is excited by a π pulse (which takes the state $|g\rangle \rightarrow -\mathrm{i}|n\rangle$ and $|n\rangle \rightarrow -\mathrm{i}|g\rangle$); again the system is allowed to dephase for τ_2, and finally the system is excited via a $\pi/2$ pulse. In summary the sequence looks as follows:

$$\pi/2 \rightarrow \tau_1 \rightarrow \pi \rightarrow \tau_2 \rightarrow \pi/2. \tag{5}$$

If decoherence are negligible then the system will exhibit a strong spin echo when the two dephasing times are equal, $\tau_1 = \tau_2$.

We simulated the effect of the spin echo sequence of equation (5) on the following system: a single ground state $|g\rangle$ atom in a sea of atoms excited to the Stark state $|n'\rangle$ or $|\psi_0\rangle = |gn'n'\cdots n'\rangle$. This is, of course, a severe approximation to a real experiment where the number of $|g\rangle$ state atoms will most certainly be greater than one, but the effect of a spin echo sequence on this simplified system should be of interest. The density of the gas was chosen so the average distance between particles would be 13.5 μm. The excitation laser coupled $|g\rangle$ to an excited Stark state $|n\rangle$, but not $|g\rangle$ to $|n'\rangle$. The two Rydberg states $|n\rangle$ and $|n'\rangle$ were chosen in a manner that allowed for the resonant exchange of energy through the dipole–dipole interaction [14]. This means that the $|n\rangle$ state on atom j can coherently hop to any atom k with $|n'\rangle$ character. If the direction of the static electric field is in the $\hat{z}$-direction then

the likelihood of hopping will be determined by $(nn'/3)^2/R_{jk}^3$, where $|n - n'| = 1$ are the principal quantum numbers of the two Stark states and R_{jk} is the distance between the two atoms. If the atoms are not allowed to move during the simulation time (i.e. the temperature is set to 0 K), then the final state of the system after a spin echo sequence can be analytically derived for the single hopper case in a sea of $N|n'\rangle$ atoms. For clarity, we first solved the two-particle case for the final state of the system,

$$\begin{aligned}
&|\psi_f\rangle \\
&= -\tfrac{1}{2}\{[\cos(V\tau_2)\,\mathrm{e}^{-\mathrm{i}(\varepsilon_g\tau_1+\varepsilon_n\tau_2)} + \cos(V\tau_1)\,\mathrm{e}^{-\mathrm{i}(\varepsilon_g\tau_2+\varepsilon_n\tau_1)}]|gn'\rangle \\
&\quad - \mathrm{i}[\sin(V\tau_2)\,\mathrm{e}^{-\mathrm{i}(\varepsilon_g\tau_1+\varepsilon_n\tau_2)} + \sin(V\tau_1)\,\mathrm{e}^{-\mathrm{i}(\varepsilon_g\tau_2+\varepsilon_n\tau_1)}]|n'g\rangle \\
&\quad + \mathrm{i}[\cos(V\tau_2)\,\mathrm{e}^{-\mathrm{i}(\varepsilon_g\tau_1+\varepsilon_n\tau_2)} - \cos(V\tau_1)\,\mathrm{e}^{-\mathrm{i}(\varepsilon_g\tau_2+\varepsilon_n\tau_1)}]|nn'\rangle \\
&\quad - [\sin(V\tau_2)\,\mathrm{e}^{-\mathrm{i}(\varepsilon_g\tau_1+\varepsilon_n\tau_2)} - \sin(V\tau_1)\,\mathrm{e}^{-\mathrm{i}(\varepsilon_g\tau_2+\varepsilon_n\tau_1)}]|n'n\rangle\} \\
&\quad \times \mathrm{e}^{-\mathrm{i}\varepsilon_{n'}(\tau_1+\tau_2)}, \qquad (6)
\end{aligned}$$

where $V = (nn'/3)^2/R_{jk}^3$ was the off-diagonal matrix element for the dipole–dipole interaction, ε_g was the energy of the ground state, ε_n was the energy of the excited Stark state $|n\rangle$ and $\varepsilon_{n'}$ was the energy of the Stark state $|n'\rangle$. When $\tau_1 = \tau_2 = \tau$, the final wavefunction is simply

$$|\psi_f\rangle = [-\cos(V\tau)|gn'\rangle + \mathrm{i}\sin(V\tau)|n'g\rangle]\,\mathrm{e}^{-\mathrm{i}(\varepsilon_g+\varepsilon_n+2\varepsilon_{n'})\tau}. \qquad (7)$$

Note that only $|gn'\rangle$ and $|n'g\rangle$ states remained, and the probability of finding an atom in the Stark state $|n\rangle$ is zero. This was a perfect spin echo. For the case of a small difference in dephasing times, $\tau_2 = \tau_1 + \Delta$ and Δ is small, this case can be simplified as well to

$$\begin{aligned}
&|\psi_f\rangle \\
&= -\left\{ \cos\left([\varepsilon_n - \varepsilon_g]\frac{\Delta}{2}\right)[\cos(V\tau_1)|gn'\rangle\mathrm{i} - \sin(V\tau_1)|n'g\rangle] \right. \\
&\quad \left. - \sin\left([\varepsilon_n - \varepsilon_g]\frac{\Delta}{2}\right)[\cos(V\tau_1)|nn'\rangle\mathrm{i} - \sin(V\tau_1)|n'n\rangle] \right\} \\
&\quad \times \mathrm{e}^{-\mathrm{i}(\varepsilon_g+\varepsilon_n+2\varepsilon_{n'})(\tau_1+\frac{\Delta}{2})}. \qquad (8)
\end{aligned}$$

Now the probability of finding a $|n\rangle$ state atom is very small and proportional to $\left[(\varepsilon_n - \varepsilon_g)\frac{\Delta}{2}\right]^2$. For the general case of one $|n\rangle$ hopper in a sea of $N|n'\rangle$, the final wavefunction is

$$\begin{aligned}
&|\psi_f\rangle \\
&= -\frac{1}{2}\left\{ \mathrm{i}\sum_{k=1}^{N+1}\left[\alpha_{ik}(\tau_2)\,\mathrm{e}^{-\mathrm{i}(\varepsilon_g\tau_1+\varepsilon_n\tau_2)} - \alpha_{ik}(\tau_2)\,\mathrm{e}^{-\mathrm{i}(\varepsilon_g\tau_2+\varepsilon_n\tau_1)}\right]|n_k\rangle \right. \\
&\quad \left. \times \sum_{k=1}^{N+1}\left[\alpha_{ik}(\tau_2)\,\mathrm{e}^{-\mathrm{i}(\varepsilon_g\tau_1+\varepsilon_n\tau_2)} + \alpha_{ik}(\tau_2)\,\mathrm{e}^{-\mathrm{i}(\varepsilon_g\tau_2+\varepsilon_n\tau_1)}\right]|g_k\rangle \right\} \\
&\quad \times \mathrm{e}^{-\mathrm{i}N\varepsilon_{n'}(\tau_1+\tau_2)}, \qquad (9)
\end{aligned}$$

where α_{ik} is the probability amplitude of finding the $|n\rangle$ state initially on atom i on atom k, $|g_k\rangle$ represents the state with $|g\rangle$ atom k and $|n_k\rangle$ represents the state with $|n\rangle$ on atom k. In the essential states model [14], the diagonal matrix elements of the Hamiltonian for resonant dipole–dipole interactions between Rydberg atoms excited to extreme Stark states are given by

$$V_{n,n';n,n'} = V_{n',n;n',n} = \left(\frac{3nn'}{2}\right)^2 \frac{1 - 3(\hat{R}_{12}\cdot\hat{z})^2}{R_{12}^3}, \qquad (10)$$

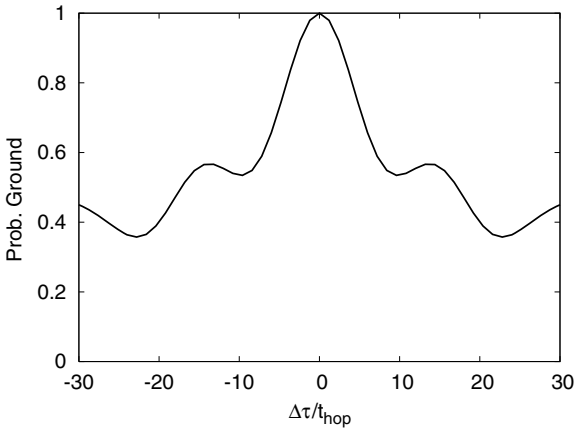

Figure 6. Spin echo signal in a 0 K gas with exactly one hopper versus difference in scaled dephasing times. When $\Delta\tau/t_{\mathrm{hop}} = 0$ a prefect signal is seen.

and the off-diagonal elements are given by

$$V_{n,n';n',n} = V_{n',n;n,n'} = \left(\frac{nn'}{3}\right)^2 \frac{1 - 3(\hat{R}_{12}\cdot\hat{z})^2}{R_{12}^3}. \qquad (11)$$

By solving for the eigenvalues and eigenvectors of the Hamiltonian given in equations (10) and (11), we found the values for the hopping amplitudes α. When $\tau_1 = \tau_2 = \tau$, the probability of finding a $|n\rangle$ state atom is zero, and the system again displays a perfect echo,

$$|\psi_f\rangle = -\sum_{k=1}^{N+1}\alpha_{ik}(\tau)\,\mathrm{e}^{-\mathrm{i}[\varepsilon_g+\varepsilon_n+2N\varepsilon_{n'}]\tau}|g_k\rangle. \qquad (12)$$

In figure 6, we plot the probability of finding no $|n\rangle$ atoms as a function of the difference in scaled dephasing time. The difference in scaled relaxation time is $\Delta\tau/t_{\mathrm{hop}}$, where $\Delta\tau = \tau_1 - \tau_2$ and $t_{\mathrm{hop}} = 0.83\,\mu$s is the time it takes for the excitation to hop the average distance between atoms: 13.5 μm. As expected, in a zero temperature gas with one hopper the system exhibits a perfect echo at $\Delta\tau/t_{\mathrm{hop}} = 0$. When the difference in dephasing times becomes large, the system becomes more and more evenly mixed.

While the previous discussion neglected the effects of temperature, by allowing the particles to exhibit thermal motion and by solving for the hopping amplitudes during every time step, we simulated a one hopper gas with an outside source of decoherence. In order to account for the time dependence of the Hamiltonian we used an exponential propagator and adjusted the position of each atom during each time step. If the time steps are kept small and the change in position is also small then the accumulated errors can be kept to a minimum. The results of our simulations are shown in figure 7 as a plot of finding zero $|n\rangle$ atoms as a function of temperature. At every temperature point the two relaxation times were exactly equal, so if thermal motion caused no decoherence then a perfect echo should be seen. If the two relaxation times between excitation pulses are just long enough to allow one hop to occur then the effect of temperature is minimal, the $|n\rangle$ state has not diffused very much. In this situation, the hopping of the $|n\rangle$ state can be limited to only the nearest neighbours; the sequence of hops to get to the final state is very simple, directly from i to k. When

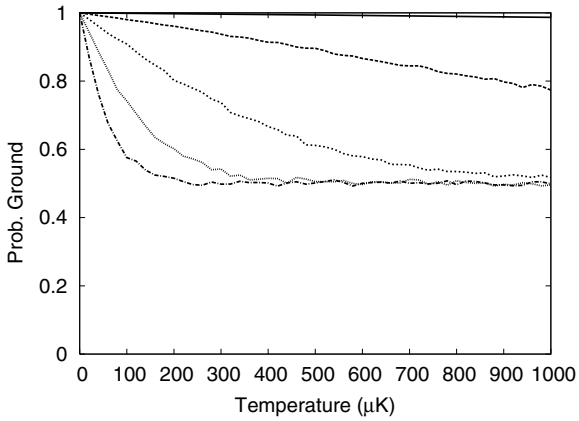

Figure 7. Spin echo signal in a gas with exactly one hopper as a function of temperature for various τ relaxation times. In each case $\tau_1 = \tau_2 = \tau$. The solid line is for $\tau = t_{\rm hop}$, the dashed line is for $\tau = 2t_{\rm hop}$, the dotted line is for $\tau = 3t_{\rm hop}$, the perforated line is for $\tau = 4t_{\rm hop}$ and the chain line is for $\tau = 5t_{\rm hop}$.

the relaxation time is increased to allow two hops, the effect of temperature is again pretty minimal well into the hundreds of microkelvins. Now the $|n\rangle$ state is limited to nearest and next-nearest neighbours and the number of possible particles it can hop to has increased by a factor of about 8, and for each one of these atoms their motion contributes to the dephasing. By increasing the relaxation time, the number of atoms involved in the hopping dynamics increases drastically for each interval of $t_{\rm hop}$. The effect of temperature is quite apparent when the number of allowed hops is 4 or above, the echo signal has effectively vanished for temperatures greater than 300 μK. As expected when the temperature gets closer to 0 K, a strong echo signal is observed, no matter how many times the excitation is allowed to hop. It should be noted that the even when the thermal motion is neglected, the dipolar force between Rydberg atoms will cause acceleration. The large mass of Rb and the shortness of $t_{\rm hop}$, however, prevent this motion from being significant. For example, after ten hops ($\sim$8.3 μs) the attractive force between two atoms initially separated by 13.5 μm will have moved them only 0.01 μm closer.

4. Conclusions

In summary, we have been able to investigate the effects of echo sequences on systems of interacting Rydberg atoms. We simulated a rotary echo signal in a dense ultra cold Rydberg gas by using the pseudoparticle many-body wavefunction approach to describe the blockade effect. Unlike simpler mean-field models, the pseudoparticle approach directly takes into account high correlation between nearby atoms, and the spatial correlations between pseudoparticles. While a mean-field model predicts the strong echo signal to be suppressed due to the strong interactions between nearby pairs of individual atoms, the pseudoparticle method prevents such short distances between Rydberg atoms. Our simulations showed a clear rotary echo signal when the sign of the excitation amplitude is switched half-way through the total excitation pulse. When all of the atoms or pseudoparticles are perfectly correlated, a perfect echo is seen. As the density between Rydberg atoms is increased the interactions between pseudoparticles are increased and the echo signal is reduced.

We also simulated a system where the sign of the entire interaction Hamiltonian is flipped, both the laser–atom interaction and the atom–atom interaction. Because the coupling between Rydberg atoms is no longer a source of dephasing, any reduction of the rotary echo signal would indicate an external source of decoherence. It might be possible to experimentally realize such a system using a weak electric field to switch the sign of van der Waals potential between two $44d_{5/2}$ Rb atoms at the same time an RF field is used to switch the phase of the excitation amplitude.

We finally examined the spin echo signal of a single coherently hopping Rydberg excitation in a gas. If thermal motion is neglected, we analytically showed that the system will display a perfect echo when the relaxation time between excitation pulses is exactly equal. Our simulations of temperature dependence on signal strength indicated that as the number of allowed hops is increased, the effect of thermal motion becomes drastically more significant. If more than four hops are allowed during each relaxation time then the temperature of the gas must be less than 300 μK for any discernable echo signal to be detected. Of interest would be further studies into the effects of multiple hoppers on the spin echo signal, and how the coherence of these systems responds to increasing temperatures.

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