We need to solve the coupled equations:

$$i\dot{a}_0 = \omega_0 a_0 + \lambda\sqrt{\frac{\Delta\omega}{\hbar}}\sum_{n=1}^{2N-1} a_n \quad \text{and} \quad i\dot{a}_n = \omega_n a_n + \lambda\sqrt{\frac{\Delta\omega}{\hbar}}\, a_0$$

Formally these equations can be written as

$$\frac{d}{dt}\vec{a} = -i\,\underline{\underline{M}}\,\vec{a}$$

From the 10th page of Chap 7 notes, $\Gamma = \frac{2\pi\lambda^2}{\hbar} \Rightarrow \lambda\sqrt{\frac{\Delta\omega}{\hbar}} = \sqrt{\frac{\Delta\omega\,\Gamma}{2\pi}}$
and $\omega_n = \omega_0 + (n-N)\Delta\omega$

The matrix has the form
with $\zeta = \sqrt{\Gamma\Delta\omega/2\pi}$

$$\underline{\underline{M}} = \begin{pmatrix} \omega_0 & \zeta & \zeta & \zeta & \cdots \\ \zeta & \omega_1 & & 0 & \\ \zeta & & \omega_2 & & \\ \zeta & 0 & & \ddots & \\ \vdots & & & & \end{pmatrix}$$

A method for numerically solving these equations is the Leap frog algorithm

$$\frac{d\vec{a}(t)}{dt} = \frac{\vec{a}(t+\delta t) - \vec{a}(t-\delta t)}{2\delta t} = -i\,\underline{\underline{M}}\,\vec{a}(t)$$

The left hand side is $O(\delta t^2)$ accurate. More importantly one can show that this approximation conserves the norm of $\vec{a}$ if $\underline{\underline{M}}$ is hermitian.

One can implement this in the following way

$$\vec{a}(2\delta t) = \vec{a}(0) - 2i\,\delta t\,\underline{\underline{M}}\,\vec{a}(\delta t)$$
$$\vec{a}(3\delta t) = \vec{a}(\delta t) - 2i\,\delta t\,\underline{\underline{M}}\,\vec{a}(2\delta t)$$

etc

Note that even time steps use the odd steps and vice versa

$$\vec{a}_e = \vec{a}_e - 2i\,\delta t\,\underline{\underline{M}}\,\vec{a}_o \qquad \vec{a}_e = \vec{a}(2\delta t)$$
$$\vec{a}_o = \vec{a}_o - 2i\,\delta t\,\underline{\underline{M}}\,\vec{a}_e \qquad \vec{a}_o = \vec{a}(3\delta t)$$

etc

Given $\vec{a}(0)$ how to get $\vec{a}(\delta t)$? Since $\underline{\underline{M}}$ is independent of time

$$\vec{a}(\delta t) = \exp[-i\,\delta t\,\underline{\underline{M}}]\,\vec{a}(0) = \vec{a}(0) - i\,\delta t\,\underline{\underline{M}}\,\vec{a}(0) - \frac{\delta t^2}{2}\underline{\underline{M}}\,\underline{\underline{M}}\,\vec{a}(0) \;\ldots$$

The evaluation of this series without the use of another complex vector is somewhat tricky. Since we only need $\vec{a}(\delta t)$ through $O(\delta t^2)$, this can be accomplished through the steps:

$$\vec{a}(\delta t) = \left(1 - i \underline{\underline{M}} \delta t - \frac{\delta t^2}{2} \underline{\underline{M}}\,\underline{\underline{M}}\right) \vec{a}(0)$$

$$= \vec{a}(0) - i \underline{\underline{M}} \delta t \left(1 - i \frac{\delta t}{2} \underline{\underline{M}}\right) \vec{a}(0)$$

The following nonintuitive algorithm accomplishes these steps

$\vec{a}_o = \vec{a}(0)$

$\vec{a}_e = \vec{a}_o - i \frac{\delta t}{2} \underline{\underline{M}}\, \vec{a}_o$

$\vec{a}_o = \vec{a}_o - i \delta t\, \underline{\underline{M}}\, \vec{a}_e$

$\vec{a}_e = \vec{a}(0)$

At this point $\vec{a}_e = \vec{a}(0)$ and $\vec{a}_o = \vec{a}(\delta t)$ + errors $O(\delta t^3)$