PHYS461, Test 2, Spring 2018

You must show work to get credit. There are integrals at the back of the book and at the bottom of the page that might be useful.

For any time dependent perturbation theory problems, only solve through 1st order.

(1) (5 pts) Estimate the lowest few rotational energy levels for a silica nanosphere of radius 50 nm. Give your answer in K.

(2) (5 pts) A particle of mass M experiences a smooth potential $V(x)$ which is 0 for $|x| > L$ and has $V(x) < E$ for all x. For $x < -L$, $\psi(x) = A\exp(-ikx)$ where A is real. At the WKB level, determine the wave function for all x. Get all amplitudes and phases correct.

(3) (5 pts) A spin-1 system has the Hamiltonian $H = \Delta E S_z/\hbar + \exp(-t^2/T^2)V_0 S_x/\hbar$ with small V_0. The system starts in spin -1 for $t \ll -T$. Determine the probability that it is in the spin 0 or the spin 1 states at $t \gg T$. (For the spin matrices see pg 195 Prob 4.53 [all of the *non-zero* off-diagonal elements for spin-1 S_x are $\hbar/\sqrt{2}$]; see board).

(4) (5 pts) A particle of mass M is in a potential $V(x) = \infty$ for $x < 0$, $V(x) = 0$ for $0 < x < L$, and $V(x) = V_0 - (x - L)F$ for $x > L$ where V_0, F are positive constants. The particle starts in the state trapped between 0 and $x \sim L$ with energy $E_0 < V_0$. As accurately as possible, estimate the lifetime of this state.

(5) (10 pts) A spin-1/2 particle has the Hamiltonian $H = (\Delta E/2)(t/T)\sigma_z$. (a) Obtain the two time dependent eigenstate solutions for this system. (b) The term $V_0\sigma_x\exp(-\alpha t^2)$ is added to H where V_0 is a small constant. If the system starts in the $-1/2$ state at $t \to -\infty$, what is the probability it will be in the $+1/2$ state at $t \to \infty$? (Hint: for part (b), you can't directly use the perturbation theory in the book but appropriately modifying Eq. [9.6] will give you the answer in a couple steps.)

(6) (10 pts) (a) As accurately as possible, determine the eigenergies, E_n, for a particle of mass M in a potential which is $V(x) = Fx$ for $x > 0$ and ∞ for $x < 0$. (b) Give the wave function in the classically allowed and unallowed regions (don't worry about normalization). (c) The wave function is $(|\psi_n\rangle + i|\psi_{n+1}\rangle)/\sqrt{2}$ at $t = 0$. What is the smallest time where the expectation value of any operator is the same as at $t = 0$. (d) In the limit of $n \gg 1$, evaluate this expression for the quantum period.

(7) (10 pts) An *electron* has the potential energy $V(x, y) = (1/2)M\omega^2(x^2 + y^2)$ and is in a laser field. The laser field gives: $\Delta V = \cos(\omega_l t)\exp(-t^2/T^2)A_l(p_x + kyp_x)$ where $\omega_l = 2\pi f_l$ with f_l the frequency of the light, $k = 2\pi/\lambda$ with λ the wavelength of the light, and A_l is a small constant. Take $\omega = 2\pi 10^{15}$ Hz. The electron starts in the ground state and the laser is weak. (a) Give the energy spacing of the states in eV. (b) What are the only states that the laser can cause a transition into with probability proportional to $|A_l|^2$? (c) What is the resonance f_l needed for each of the allowed cases? (d) Calculate the probability for each transition versus ω_l. (e) What is the *relative probability* for each transition when that transition is perfectly on resonance? Given numerical values.

Possibly useful integral $\int_{-\infty}^{\infty}\exp(iax)\exp(-x^2)dx = \sqrt{\pi}\exp(-a^2/4)$