# Physics 310 - First Midterm - October 12, 2006

## *Instructions*

- This exam has three pages including this cover sheet.
- There are four questions on this exam.
- The questions are to be answered in the blue exam booklets.
- Write your name on every exam booklet you use.
- No notes, text books, crib sheets, *etc.* are to be used when writing this exam.
- You will not need a calculator for this exam.
- The exam begins at 9:30 am and must be turned in by 11:30 am.
- No guinea-pigs were harmed in the making of this exam.

**1.** A particle travels on a spiral path that is described in polar coordinates by the variables:

$$\begin{aligned} r(t) &= be^{kt} \\ \theta(t) &= ct \end{aligned}$$

where $b$, $k$ and $c$ are constants.

(a) Calculate the velocity $\boldsymbol{v} = d\boldsymbol{r}/dt$.

(b) Calculate the magnitude of the velocity, $v = |\boldsymbol{v}|$.

(c) Calculate the acceleration $\boldsymbol{a} = d\boldsymbol{v}/dt$.

(d) Calculate the magnitude of the acceleration, $a = |\boldsymbol{a}|$.

**2.** A particle of mass $m$ is shot horizontally into a liquid that exerts a force that is proportional to the velocity of the particle: $F_{\text{drag}} = -cv$.

(a) If the particle has initial velocity $v_0$ when it enters the liquid at $t = 0$, calculate its horizontal distance, $x(t)$, as a function of time.

(b) What is the maximum distance that it can reach in the liquid?

**3.** Show that any "central force" is conservative. That is, show that a force $\boldsymbol{F}(\boldsymbol{r})$ is conservative when it is of the form

$$\boldsymbol{F}(\boldsymbol{r}) = f(r)\hat{\boldsymbol{r}}$$

where $r = \sqrt{x^2 + y^2 + z^2}$ is the distance from the origin of the coordinate system. Remember that you can write $\hat{\boldsymbol{r}} = \boldsymbol{r}/r$.

**4.** A merry-go-round rotates with constant angular velocity $\boldsymbol{\omega} = \omega\hat{\boldsymbol{k}}$. "Chipper", a grade 4 class guinea-pig at the Miller Elementary School in Lafayette, starts at the center at $t = 0$ and crawls with a constant speed in the radial direction so that its position is given by $\boldsymbol{r}(t) = vt\hat{\boldsymbol{r}}$. Here, $\hat{\boldsymbol{r}}$ is the radial unit vector that rotates with the merry-go-round so that it is constant when viewed in this non-inertial reference frame.

(a) If the mass of the guinea-pig is $m$, calculate the inertial, or "fictitious" forces that appear to act on the animal when viewed from the rotating reference frame of the merry-go-round.

(b) Calculate the magnitude of the sum of the intertial forces as a function of time.

(c) If the coefficient of static friction between Chipper and the merry-go-round is $\mu$, how far will Chipper crawl before he slips?

Remember that

$$\boldsymbol{a'} \;=\; \boldsymbol{a} - \dot{\boldsymbol{\omega}} \times \boldsymbol{r'} - 2\boldsymbol{\omega} \times \boldsymbol{v'} - \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r'}).$$

where $\boldsymbol{a}$ is the acceleration in an inertial reference frame and the primed quantities are measured in a reference frame that rotates with angular velocity $\boldsymbol{\omega}$. Also,

$$\begin{aligned} \hat{\boldsymbol{r}} \times \hat{\boldsymbol{\theta}} &= \hat{\boldsymbol{k}} \\ \hat{\boldsymbol{\theta}} \times \hat{\boldsymbol{k}} &= \hat{\boldsymbol{r}} \\ \hat{\boldsymbol{k}} \times \hat{\boldsymbol{r}} &= \hat{\boldsymbol{\theta}} \end{aligned}$$