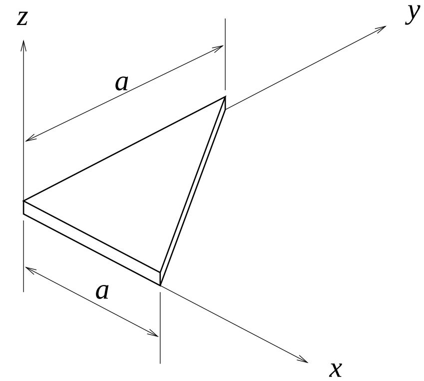
Physics 310 - Assignment #5 - Due December 7^{th}

1. Consider the thin triangular lamina of mass M with dimensions shown below:



(a) Calculate the inertia tensor of the lamina about the origin when it is in the orientation shown.

(b) Determine the principal axes and the moments of inertia when the lamina is rotating around each principal axis.

2. A small ball of putty of mass m moving with velocity $\boldsymbol{v} = v\hat{\boldsymbol{\imath}}$ hits the thin triangular lamina described in question 1, and sticks to it at the point $\boldsymbol{x} = a\hat{\boldsymbol{\jmath}}$.

(a) Calculate the position of the center of mass of the system after the collision.

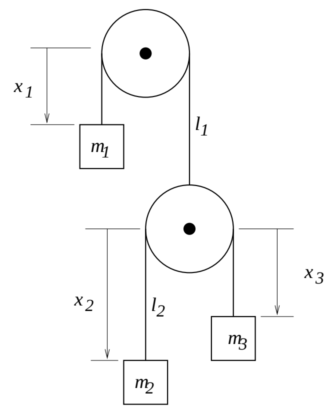
(b) If the lamina is initially at rest, calculate the velocity of the center of mass of the system after the collision.

(c) Calculate the angular velocity about the center of mass after the collision.

3. The thin triangular lamina described in question 1 is rotating with angular velocity ω about the x-axis. Use Euler's equations to calculate the magnitude and direction of the torque that is required to keep the lamina rotating about this fixed axis, expressing its components in the rotating coordinate system in which the lamina is fixed.

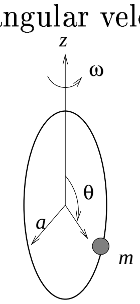
4. A rigid body of arbitrary shape rotates freely under zero torque. By means of Euler's equations, show that both the rotational kinetic energy and the magnitude of the angular momentum are constant. *Hint: for $\boldsymbol{N} = 0$, multiply Euler's equations by ω_1, ω_2 and ω_3, respectively, and add the three equations. The result indicates constancy of kinetic energy. Next, multiply by $I_1\omega_1$, $I_2\omega_2$ and $I_3\omega_3$, respectively and add. The result shows that L^2 is constant.*

5. Use the method of Lagrange multipliers to find the equations of motion for the three masses and the tensions in the strings of the "Double Atwood Machine":



in which the strings have length l_1 and l_2 and the pulleys are assumed to be massless.

6. A bead of mass m, which is acted on by gravity, slides on a thin, circular hoop of radius a that spins about the z axis with constant angular velocity ω as shown:



Find the equation of motion of the bead expressed in terms of the angle θ. What is the period of small oscillations about its equilibrium position?