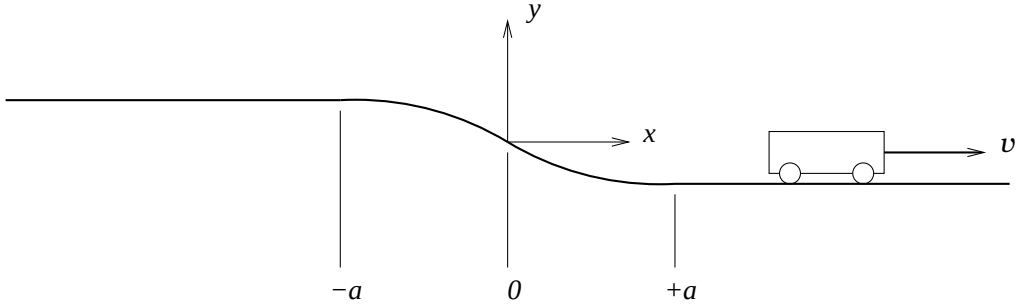
Physics 310 - Assignment #3 - Due October 19^{th}

1. A car on a roller coaster travels along a path, as shown, that is described by the equation

$$\begin{aligned} X(t) &= vt \\ Y(t) &= \frac{h}{2}\left(\frac{3X(t)}{a} - \frac{X(t)^3}{a^3}\right) \end{aligned}$$

when $-a < X(t) < a$, with $Y(t) = \pm h$ when $X(t) < -a$ and $X(t) > a$, respectively.



A person riding in the car is drinking a cup of tea in which a tea bag of mass m is floating with its bouancy perfectly cancelling the force of gravity. When in motion through the liquid, the tea bag is subjected to a damping force that is proportional to its velocity in the liquid.

(a) Write the differential equation that the position, y', of the tea bag must satisfy when viewed in the non-inertial reference frame that moves with the car. Identify which forces are real, and which are inertial forces caused by the acceleration of the car. Consider the time intervals $t < -a/v$, $-a/v < t < a/v$ and $t > a/v$ as separate cases.

(b) Sketch a graph showing the behavior of $y'(t)$ when the tea bag is initially at $y' = 0$ and is stationary so that $\dot{y}' = 0$, when $t < -a/v$. Don't solve the differential equation, but instead just indicate the expected behavior.

2. A bullet is fired straight up with initial speed v_0'. Assuming g is constant and ignoring air resistance, show that the bullet will hit the ground west of the initial point of upward motion by an amount $4\omega v_0'^3 \cos\lambda/3g^2$, where λ is the lattitude and ω is the Earth's angular velocity.

3. The force on a charged particle in an electric field $\boldsymbol{E}$ and a magnetic field $\boldsymbol{B}$ is given by

$$\boldsymbol{F} \;=\; q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$$

in an inertial system, where q is the charge and $\boldsymbol{v}$ is the velocity of theparticle in the inertial system. Show that the differential equation of motion referred to a rotating coordinate system with angular velocity $\boldsymbol{\omega} = -(q/2m)\boldsymbol{B}$ is, for small $\boldsymbol{B}$,

$$m\ddot{\boldsymbol{r}}' \;=\; q\boldsymbol{E}.$$

4. An ant crawls with constant speed in a circular path of radius b on a photograph turntable with constant angular speed ω. The circular path is concentric with the center of the turntable. If the mass of the ant is m and the coefficient of static friction with the surface of the turntable is μ_s, how fast relative to the turntable can the ant crawl before it starts to slip if it goes (a) in the direction of rotation and (b) opposite to the direction of rotation?