Physics 310 - Assignment #2 - Due September 28^{th}

1. A particle is subjected to a force that is described by a potential

$$V(x) = ax + b/x$$

where x is always positive.
(a) Find the equilibrium position, x_0, at which the force vanishes.
(b) Find the effective spring constant, k, that would describe the force in the limit where the amplitude of oscillations about the equilibrium position is small.

2. The force acting on a particle of mass m is given by

$$F = kvx$$

in which k is a positive constant. The particle passes through the origin with speed v_0 at time $t = 0$. Find $x(t)$ and explicitly show that it is a solution to the differential equation $F = m\ddot{x}$ with the given initial conditions.

3. The suspension on a car of mass m consists of springs and shock absorbers. In a simplified model for the suspension, the springs produce a restoring force proportional to $-kx$ and the shock absorbers produce a damping force proportional to $-c\dot{x}$, where x is the height that the car bounced from its equilibrium position.
(a) If $m = 1000$ kg and $k = 1.5 \times 10^5$ N/m, what value of c will result in critically damping of oscillations?
(b) Suppose that two shock absorbers broke, resulting in a value of c that was only half the value determined in part (a). What frequency of a periodic driving force would produce the maximal amplitude of oscillations?

4. A "stiff" spring is modelled by a force that is of the form

$$F(x) = -kx - \lambda x^3$$

where λ is a small positive constant. A proposed solution to the differential equation $F(x) = m\ddot{x}$ is

$$x(t) = A\cos\omega t + \lambda B \cos 3\omega t$$

with the initial condition $x(0) = A_{\max} = A + \mathcal{O}(\lambda)$. Find an approximate expression for the frequency of oscillation, $f = \omega/2\pi$, that is valid up to $\mathcal{O}(\lambda)$.

5. *(Fowles & Cassiday, 4.19)* An atom is situated in a simple cubic crystal lattice. If the potential energy of interaction between any two atoms is of the form $cr^{-\alpha}$, where c and α are constants and r is the distance between the two atoms, show that the total energy of interaction of a given atom with its six nearest neighbors is approximately that of the three-dimensional harmonic oscillator potential

$$V \approx A + B(x^2 + y^2 + z^2)$$

where A and B are constants.

6. *(Fowles & Cassiday, 4.22)* A bead slides on a smooth rigid wire bent into the form of a circular loop of radius b. If the plane of the loop is veritcal, and if the bead starts from rest at a point that is level with the center of the loop, find the speed of the bead at the bottom and the reaction of the wire on the bead at that point.

7. A bead slides on a smooth rigid wire bent into the form of a *parabola* that is described by the equation

$$z = \frac{1}{2}\left(\frac{x^2}{v^2} - v^2\right)$$

where v is a constant. If the bead starts from a height z_0, find the speed of the bead at the bottom and the reaction of the wire on the bead at that point.